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Scientific paper

Design of Rigid Pile Caps through an Iterative Strut-and-Tie Model

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Abstract

The aim of this work is to present a strut-and-tie model for design of reinforced concrete pile caps. The model considers both failure by crushing of the compressed struts and by yielding of the tie reinforcement. Unlike some traditional models, crushing of the compressed concrete is not checked at the section in direct contact with the column base (column/pile cap interface). In this work, crushing of concrete is verified in a section at a certain depth inside the pile cap. Thus, this verification is replaced by determining the height of the nodal zone at the top of the pile cap required not to cause crushing of the struts. An iterative algorithm is used for this purpose. Comparison with a large number of experimental results available in the literature demonstrates the effectiveness of the proposed model for the design of concrete pile caps. Numerical examples of practical use of the model are also presented.

1. Introduction

Codes for reinforced concrete structures consider two different methods for design of pile caps. In the first method, the pile cap is analyzed as a beam or a slab supported on piles. The main reinforcement is calculated as in a bending problem, for the bending moment in a reference section located in the column. Shear strength is checked using the same criterion as in beams. Punching shear is verified as in slabs (ACI 318-14 Building Code 2014; Japanese Code JSCE 2010; Spanish Code EHE 2011). Usually, this sectional method is employed for flexible pile caps, where the distance between the axis of any pile to the column face is more than twice the height of the pile cap.

In order to avoid the necessity of one-way shear reinforcement, shear in a reference section is limited by the same formula used for thin slabs. The shear resistance depends on the compressive strength of concrete and reinforcement ratio (ACI 318-14; JSCE 2010). Some design codes (EHE 2011; Eurocode EC2 2014) also consider the slab thickness in the evaluation of the shear resistance. Usually, the reference section used to calculate the factored shear force is taken at a distance d from the column face, where d is the effective depth of the pile cap.

Failure by punching shear is checked in a control perimeter located at a distance $d/2$ from the column face (ACI 318-14; FIB Model Code 2010), or at a distance $2d$ (EHE 2011; EC2 2014). There is a lack of uniformity with respect to the location of the control perimeter as well as the value of punching shear resistance. Additional checks on the perimeter of the column cross section and around the piles may also be needed.

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In the second method, pile caps are designed using a model of spatial truss, also called strut-and-tie model (Adebar and Zhou 1996; Brown *et al*. 2006; Chantelot and Mathern 2010). The verifications aim to limit the compressive stresses in the concrete struts so as to prevent a brittle failure. If the struts are idealized as prismatic or uniformly tapered compression members (ACI 318-14), it is usually sufficient to limit the compressive stresses in the nodes of the truss, located near the piles and near the column. Then, the tie reinforcement is calculated. This method is employed for rigid pile caps, where the distance between the axis of any pile to the column face is less than twice the height of the pile cap (ACI 318-14; EHE 2011).

Figure 1 shows the strut-and-tie model usually employed for two-pile caps. The column is subjected to a centered load and has a rectangular cross section.

In the classical model shown in **Fig. 1**, the two struts go from the column base, on the top of the pile cap, towards the axes of piles at the reinforcement level. The design load N_d is distributed equally to the two struts, being applied at a distance $0.25a$ from the column axis, where a is the dimension of the column cross section in the direction of the piles.

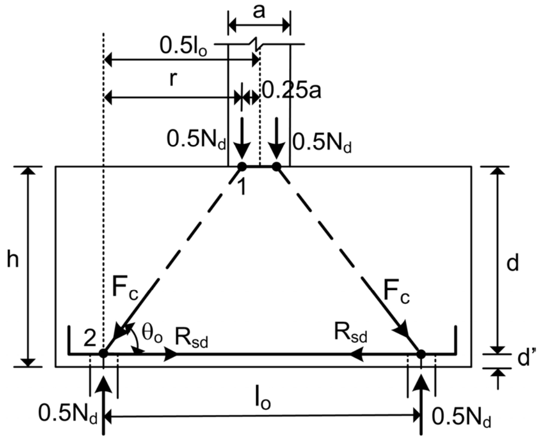


Fig. 1 Classical strut-and-tie model for two-pile caps.

The inclination of the compressive struts is calculated with the relationship $\tan\theta_o = d/r$, where $r = 0.5l_o - 0.25a$ and l_o is the distance between the piles axes. The compressive force on each strut is $F_c = 0.5N_d/\sin\theta_o$ and the force on the tie is $R_{sd} = 0.5N_d \cot\theta_o$. The tie reinforcement area is calculated as $A_s = R_{sd}/f_{yd}$, where f_{yd} is the design yield strength of the steel.

Compressive stresses in the struts are evaluated on nodes 1 and 2, below the column and over the piles, respectively. These stresses are given by

$$\sigma_{c1} = \frac{N_d}{A_c \sin^2\theta_o} \quad (1)$$

$$\sigma_{c2} = \frac{N_d}{2A_p \sin^2\theta_o} \quad (2)$$

where A_c is the area of the column cross section and A_p is the area of the pile cross section.

Finally, these stresses are compared with a concrete effective strength $f_{cd,ef}$ to ensure safety against crushing of the struts (Oliveira *et al.* 2014; Munhoz 2014). This effective strength depends on the number of piles (Blévot and Frémy 1967). It is usually adopted $f_{cd,ef} = 1.2f_{cd}$ for two pile caps, $f_{cd,ef} = 1.5f_{cd}$ for three pile caps, and $f_{cd,ef} = 1.8f_{cd}$ for four pile caps (Munhoz 2014), where f_{cd} is the design value of the uniaxial compressive strength of concrete.

This traditional model, despite being based on tests by Blévot and Frémy (1967), has two inconsistencies. The first is due to the calculation of the stress σ_{c2} using equation (2), which considers the actual area A_p of the pile cross section. This stress would be correct if the distance d' between the reinforcement axis and the bottom face of the pile cap was null. However, as $d' > 0$ there is a dispersion of the contact stresses up to the level of reinforcement, with a consequent reduction of σ_{c2}, as shown in **Fig. 4**. Thus, a way to reduce the strut stress consists of distributing the tie reinforcement in several layers, which increases d'. Of course, this reduces the effective depth d, demanding greater steel area for the tie.

The other inconsistency refers to the stress σ_{c1} at the column base. This stress is correct if the column reinforcement is not extended into the pile cap, which is not the procedure used in practice. However, even in this unusual case, there will be an increase in bearing strength.

Since the column reinforcement bars penetrates until the bottom of the pile cap, or dowel bars are used, the design load is progressively transferred by adherence and, mainly, through the amplification of the compressed area inside the pile cap (Fusco 1995). Indeed, at the column/pile cap interface, only the load $N_{dc} = N_d - N_{ds}$ is transferred immediately to the concrete of the pile cap, where N_{ds} is resisted by the column reinforcement. Failure due to bearing stress only occurs if the concrete of the pile cap has a much lower resistance that the concrete of the column.

Equation (1) imposes a strong restriction in the value of the maximum load of the column. For example, for two pile caps, one must have $N_d \le 1.2A_c f_{cd} \sin^2\theta_o$. For $\theta_o = 45^o$ results $N_d \le 0.6A_c f_{cd}$, regardless of the column reinforcement ratio. This causes a major limitation in the column design.

This study aims to present an alternative model for design of rigid pile caps, eliminating the inconsistencies above mentioned. The central idea of the model follows the previous work of Fusco (1995). However, this paper presents an innovative way of calculating the depth of the nodal zone at the base of the column. The safety of the proposed model is demonstrated through the analysis of a large number of pile caps tested by other authors. Due to the importance of this structural element it is recommended that the design be conservative, which is the case of the proposed model. The study is limited to cases of static loading. The struts are idealized as prismatic or uniformly tapered compression members. Bottle-shaped struts are not considered.

2. Proposed model for design of pile caps

In the proposed model of this study, it is considered that the struts converge to a horizontal plane situated at a distance x from the top of the pile cap. In this plane, the vertical stress σ_{vd} has been reduced enough not to cause crushing of the struts. The compressive stress σ_c in the strut near the top of the pile cap is given by $\sigma_c = \sigma_{vd}/\sin^2\theta$, where θ is the strut inclination. The inclination angle of the strut must satisfy the relationship $\tan\theta \ge 1/2$, in other words $\theta \ge 26.6°$. ACI Building Code 2014 (ACI 318-14) requires $\theta \ge 25°$. The height of the pile cap is chosen to ensure this minimum inclination for the concrete struts.

In order to avoid crushing of the struts near the top of the pile cap, it is necessary to limit $\sigma_c \le f_{cd1}$, where f_{cd1} is the design compressive strength of concrete in this zone. Therefore, the intended horizontal plane is one where $\sigma_{vd} \le \sin^2\theta f_{cd1}$, as shown in **Fig. 2**.

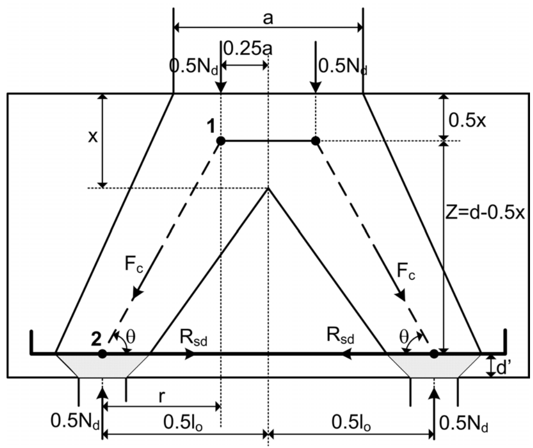


Fig. 4 Proposed strut-and-tie model for two-pile caps.

As indicated in **Fig. 2**, the region 1 with depth equal to x, located under the column, is nothing more than an extension of the column within the pile cap. In this region the column has an enlarged base. Due to the confinement provided by the large concrete cover, concrete is subjected to a triaxial compression state (for pile caps on several piles), or a biaxial compression state (for two-pile caps). Thus, there is a significant increase in the uniaxial compressive strength f_c, with no risk of crushing in this zone even if the concrete of the pile cap has a strength somewhat lower than the concrete of the column.

Usually, region 1 is referred to as CCC nodal zone because in a two-dimensional problem it receives three compressive forces. Several design codes provide limits to the compressive stress in this nodal zone. For an unconfined node, the Eurocode EC2 (2014) adopts $f_{cd1} = 1.0(1 - f_{ck}/250) f_{cd}$, where f_{ck} is the characteristic strength in MPa and f_{cd} is the uniaxial design compressive strength of concrete. According to this equation, it is found that $f_{cd1} \geq 0.85 f_{cd}$ if $f_{ck} \leq 37.5$ MPa. The limit $0.85 f_{cd}$ for the compressive stress is also adopted by ACI (2014), JSCE (2010) and Canadian Code CSA (2014). For triaxially compressed nodes the value of f_{cd1} can be increased. So, in this work it is adopted $f_{cd1} = 0.85 f_{cd}$ for two-pile caps and $f_{cd1} = f_{cd}$ for pile caps on more than two piles, which is conservative as shown by comparison with experimental results.

Figure 3 shows magnified areas under the column at a depth x from the top of the pile cap. The column has rectangular cross section with sides a and b. For two-pile caps, it is considered that the amplification only occurs in the direction of the piles. As a simplification for pile caps on three or more piles, it is assumed an expansion of the area in both directions as shown in **Fig. 3**.

Figure 4 shows the proposed strut-and-tie model.

The inclination of the struts is given by $\tan\theta = (d - 0.5x)/r$, where $r = 0.5 l_o - 0.25a$. In the traditional model shown in **Fig. 1**, the strut inclination is $\tan\theta_o = d/r$, with the strut going to the top of the pile cap. Therefore, there is the relationship

$$\tan\theta = \tan\theta_o \left(1 - \frac{0.5x}{d}\right) \tag{3}$$

Thus, for a given value of x it is possible to calculate the inclination of the compressive struts and the magnified area A_v under the column as shown in **Fig. 3**. The vertical stress in the magnified area is given by $\sigma_{vd} = N_d / A_v$ and is a function of x. Imposing the condition $\sigma_{vd} = \sin^2\theta f_{cd1}$, x can be obtained through an iterative process.

Defining the relative normal force

$$\nu = \frac{N_d}{abf_{cd}} \tag{4}$$

and imposing the condition $\sigma_{vd} = \sin^2\theta f_{cd1}$, the following expressions are obtained for x, according to the number of piles.

<u>Two-pile caps</u> ($f_{cd1} = 0.85 f_{cd}$):

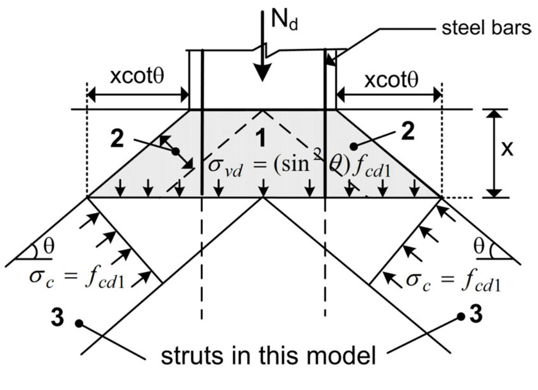
The magnified area is given by

$$A_v = b\left(a + 2x\cot\theta\right) \tag{5}$$

as shown in **Fig. 3**.

Considering the equations (4) and (5), the vertical stress $\sigma_{vd} = N_d / A_v$ in the magnified area is given by

$$\sigma_{vd} = \frac{\nu abf_{cd}}{b\left(a + 2x\cot\theta\right)} \tag{6}$$



1 : reinforced concrete; biaxial or triaxial compression

2 : struts in the classical model

3 : struts in the proposed model

Fig. 2 Verification of the concrete struts under the column.

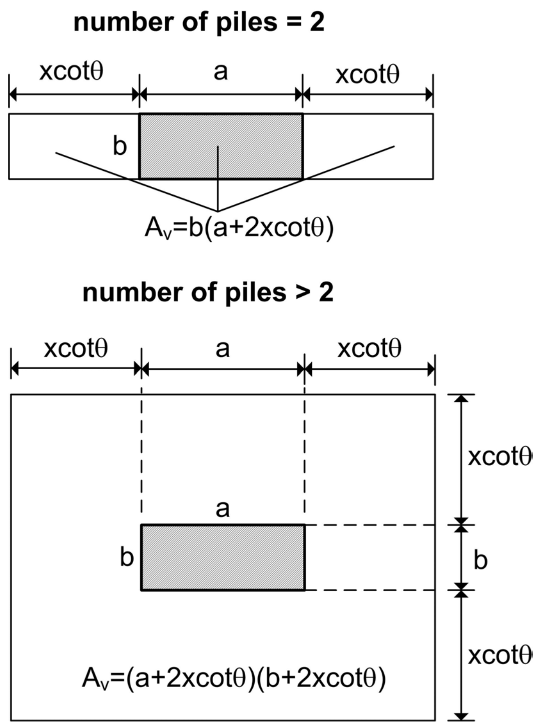


Fig. 3 Magnified area under the column.

Imposing the condition $\sigma_{vd} = \sin^2\theta f_{cd1}$, where $f_{cd1} = 0.85 f_{cd}$, it results

$$\frac{x}{a} = \frac{\nu - 0.85\sin^2\theta}{1.7 \quad \sin\theta\cos\theta} \tag{7}$$

Pile caps with more than two piles ($f_{cd1} = f_{cd}$):

The magnified area is given by

$$A_v = (a + 2x\cot\theta)(b + 2x\cot\theta) \tag{8}$$

as shown in **Fig. 3**.

Considering the equations (4) and (8), the vertical stress $\sigma_{vd} = N_d / A_v$ in the magnified area is given by

$$\sigma_{vd} = \frac{\nu a b f_{cd}}{(a + 2x\cot\theta)(b + 2x\cot\theta)} \tag{9}$$

Imposing the condition $\sigma_{vd} = \sin^2\theta\, f_{cd1}$, where $f_{cd1} = f_{cd}$, it results

$$\frac{x}{a} = \frac{-(1+\lambda)\sin\theta + \sqrt{(1+\lambda)^2\sin^2\theta + 4\lambda(\nu - \sin^2\theta)}}{4\cos\theta} \tag{10}$$

where $\lambda = b/a$ is the ratio between the sides of the column cross section.

These equations can only be solved iteratively. For this purpose, the following procedure is adopted:

Step 1: Assume $x = 0$ and calculate the angle $\theta = \theta_o$ through the relationship $\tan\theta_o = d/r$.

Step 2: Compute x by means of equations (7) or (10), as appropriate. If $x \le 0$, the solution is $x = 0$, indicating that the struts can converge to the top of the pile cap, as in the classical model. If $x > 0$, go to the next step.

Step 3: With the value of x obtained in the previous step, compute a new angle θ by means of the equation (3). With this value θ, calculate the new value of x through the equations (7) or (10). Proceed iteratively until convergence of x. The adopted convergence criterion is: $\left|x_j - x_{j-1}\right| / x_j < 0.01$, where x_{j-1} and x_j are the values obtained in two successive iterations.

To ensure a minimum ductility and prevent brittle failure, the depth x of the horizontal plane obtained from equations (7) and (10) should be limited. Thus, the relationship x/d is restricted to the values $x/d \le 0.45$ (for $f_{ck} \le 35$ MPa) and $x/d \le 0.35$ (for $f_{ck} > 35$ MPa), according to CEB-FIP Model Code (1993) recommendations. Similarly, the angle of inclination of the struts is limited to $\theta \ge 26.6°$. If these restrictions are not met, the effective depth d of the pile cap and/or the dimensions of the column cross section must be increased. The same should be done if the iterative process does not converge.

To avoid the iterative process, it may be adopted a minimum value for the angle θ, computing x by means of equations (7) or (10). Fusco (1995), for example, adopts $\theta = 26.6°$ as the minimum strut inclination. This procedure simplifies the structural design but it can be uneconomical, particularly, for two-pile caps. Other authors (Jimenez Montoya et. al 2000; Calavera 2000) consider a fixed value for x, such as $x = 0.30d$, and the strut inclination θ is obtained from the equation (3). This value of x can be excessive if the column is not heavily loaded, and the solution is uneconomical. Anyway, the proposed iterative process involves simple calculations as well as converges very quickly, being the recommended solution.

If the column transmits a bending moment to the pile cap, the relative normal force ν must be calculated for an equivalent load $N_{de} > N_d$. As a simplification, one can consider that N_{de} is equal to the number of piles multiplied by the design reaction of the most loaded pile (Santos *et al*. 2015).

Once x is known, the lever arm $Z = d - 0.5x$ is definied as shown in **Fig. 4**. The tie steel area is calculated as $A_s = R_{sd} / f_{yd}$, where $R_{sd} = 0.5 N_d \cot\theta$ and $\cot\theta = (0.5 l_o - 0.25a)/Z$. Therefore,

$$A_s = \frac{0.5 N_d (0.5 l_o - 0.25a)}{Z f_{yd}} \tag{11}$$

This equation may be written as

$$A_s = \frac{M_d}{Z f_{yd}} \tag{12}$$

where M_d is the design bending moment in a reference section located at a distance $0.25a$ behind the column face caused by the pile reaction.

The pile reactions are obtained considering the pile cap as a rigid body and each pile is modeled as a spring element. If the piles are loaded unequally due to the eccentricity of the force N_d, it must be considered the one that causes the largest value of M_d in the reference section. The model may be used to calculate the reinforcement of pile caps supported on several piles. In order to do this, just calculate the reaction of each pile and determine the maximum bending moment in the reference section. This bending moment is calculated considering the reactions of all piles located at the same side of the analyzed section.

Figure 5 indicates the sections for calculation of the reinforcement in two orthogonal directions, for a pile cap with many piles.

The reinforcement in direction 1 is calculated for the bending moment in the section S1, caused by the reactions of all piles located on the right of this section. If

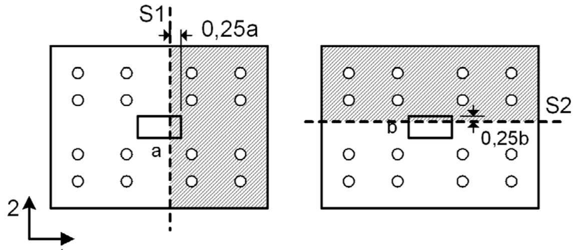


Fig. 5 Reference sections for calculation of the reinforcement.

the piles on the left cause a higher bending moment, one should consider the section S1 located in this side. The reinforcement in direction 2 is calculated for the bending moment in the section S2, in a similar way. The reinforcement in each direction should be concentrated in the alignment of the piles. Additional reinforcement placed between the piles may be required.

The strut stress at node 2 on the pile is given by

$$\sigma_{c2} = \frac{F_{de}}{A_{amp}\sin^2\theta} \tag{13}$$

where F_{de} is the design pile reaction, $A_{amp} = kA_p$ is the amplified area on the pile and k is a factor that takes into account the spreading of the contact stresses to the centroid of the reinforcement.

For two-pile caps subjected to a centered load as shown in **Fig. 4**, $F_{de} = 0.5N_d$. If the piles are unequally loaded, one should determine the node where σ_{c2} is maximum.

It is assumed that the contact stresses on the piles spread at 45 degree angles in all directions, according to the recommendations of EC2 (2014). Thus, the coefficient k is given by

$$k = \left(1 + \frac{2d'}{\phi_p}\right)^2 \tag{14}$$

where ϕ_p is the diameter of the pile cross section.

For two-pile caps, the spreading occurs only in the direction of the piles and $k = 1 + 2d'/\phi_p$. If it is necessary to consider the bidirectional amplification given in equation (14) to avoid crushing of the strut, transverse reinforcement is required to restraint vertical splitting on the pile. This can be modeled using a transverse strut-and-tie model. The area of this transverse reinforcement is given by $A_{st} = 0.25F_{de}/f_{yd}$. This reinforcement is not necessary for pile caps with more than two piles.

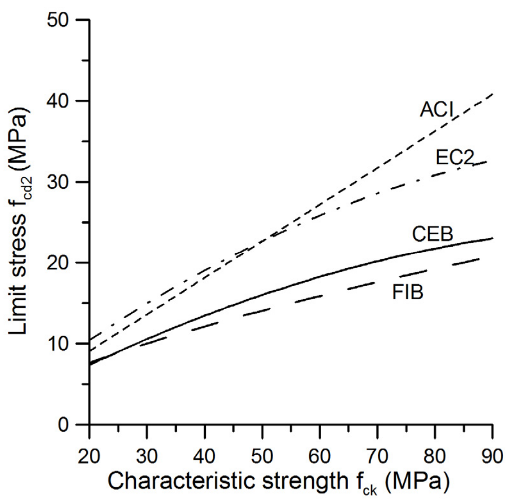


Fig. 6 Strength of CCT nodal zone according to different design codes.

For piles of square section, ϕ_p is the side of the cross section. In all cases, it is recommended to limit $k \le 4$.

The nodal zone over the pile is referred to as CCT nodal zone because it receives two struts and one tie. In order to avoid crushing of the struts it is necessary to limit $\sigma_{c2} \le f_{cd2}$. Here there is no consensus about the limit to the compressive stress f_{cd2}, as shown below:

ACI 318-14 (2014): $f_{cd2} = 0.68 f_{cd}$

JSCE Code (2010): $f_{cd2} = 0.68 f_{cd}$

Canadian Code CSA (2014): $f_{cd2} = 0.70 f_{cd}$

Spanish Code EHE (2011): $f_{cd2} = 0.70 f_{cd}$

Eurocode EC2 (2014): $f_{cd2} = 0.85\left(1 - \frac{f_{ck}}{250}\right) f_{cd}$

FIB Model Code (2010): $f_{cd2} = 0.50\left(\frac{30}{f_{ck}}\right)^{1/3} f_{cd}$

CEB-FIP Model Code (1993): $f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right) f_{cd}$

Figure 6 shows the variation of f_{cd2} with the characteristic strength f_{ck} according to these standards, considering $f_{cd} = f_{ck}/1.5$. Expressions of EHE, JSCE and Canadian Code were not plotted because they are similar to the ACI equation. As can be observed, expressions of CEB and FIB provide very similar values to f_{cd2}. Equations of ACI and EC2 provide the highest values. Therefore, conservatively, it is recommended to adopt the expressions of FIB or CEB.

Thus, in this work it is adopted

$$f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right) f_{cd} \tag{15}$$

according to CEB-FIP Model Code (1993) recommendations.

3. Comparison with experimental results

In order to demonstrate the validity of the proposed model, 138 pile caps tested by other authors have been analized. These experimental results include 37 two-pile caps, 21 three-pile caps and 80 four-pile caps. All pile caps were subjected to a centered load. The columns have square or rectangular cross section. The pile sections can be square, rectangular or circular. Concrete compressive strength f_c based on tests of cylinders varies from 13.2 MPa to 49.3 MPa.

Table 1 shows summary information about the pile caps. Full details may be obtained in references listed in the table. For four-pile caps, the complete data may be obtained in Souza *et al*. (2009).

When carrying the structural design, the design load N_d is given by $N_d = \gamma_f N_k$, where N_k is the characteristic load and $\gamma_f > 1$ is a partial safety factor. Design compressive strength of concrete is $f_{cd} = f_{ck}/\gamma_c$, where f_{ck} is the characteristic strength and $\gamma_c > 1$ is other partial safety factor. Finally, design yield strength of

Table 1 Pile caps used for checking the model.

Author (Year)	Number of pile caps	Number of piles	Pile section	f_c MPa minimum	f_c MPa maximum
Munhoz (2014)	11	2	Square	32.8	33.9
Mautoni (1972)	20	2	rectangular	19.5	32.3
Blévot (1967)	6	2	Square	23.6	47.0
Blévot (1967)	12	3	Square	17.7	37.4
Miguel (2000)	9	3	Circular	24.5	40.3
Blévot (1967)	31	4	Square	13.2	49.3
Clarke (1973)	13	4	Circular	22.5	43.7
Suzuki (1998)	19	4	Square	18.9	30.9
Suzuki (1999)	17	4	Square	25.6	30.9

reinforcement is $f_{yd} = f_{yk}/\gamma_s$, where f_{yk} is the characteristic yield strength and $\gamma_s > 1$ is a third partial safety factor.

When comparing experimental results with those obtained through a theoretical model, the partial safety factors should be considered equal to 1.0. Thus, design strength and characteristic strength of materials are considered equal to their experimental values. So, when using the theoretical model, it should be adopted $f_{ck} = f_c$; $f_{cd} = f_c$; $f_{yd} = f_y$. The purpose of this analysis is to compare the experimental failure load $P_{u,\exp}$ with the theoretical failure load $P_{u,teo}$ without taking into account the partial safety factors. The ratio $R = P_{u,teo}/P_{u,\exp}$ is a measure of validity of the model. If $R \le 1$, it means that the model is safe because it provides a smaller failure load than that observed experimentally. That is, the theoretical model underestimates the structural resistance, as desirable. Conversely, $R > 1$ indicates that the model overestimates the structural load capacity. In this case, the model error may be covered by the partial safety factors if R is not much greater than 1.

The model provides two failure modes: crushing of the compressive struts on the piles and yielding of the reinforcement. Furthermore, to avoid concrete crushing at the column base, it is considered the lever arm $Z = d - 0.5x$, with x being the depth of the horizontal plane where the vertical stress σ_{vd} is equal to $\sin^2\theta \;\; f_{cd1}$.

To determine the theoretical failure load $P_{u,teo}$, is employed an incremental process for the load N_d. For each value of N_d, it is determined the depth x of the horizontal plane with the iterative algorithm presented previously. Then it is verified the compressive stress in the strut on the pile with use of the equations (13) to (15). Finally, the tensile force in the tie, R_{sd}, is compared with its strength $A_s f_{yd}$. If the rupture does not occur for any of these two failure modes, the load is increased to find $P_{u,teo}$.

The amplified area on the pile is given by $A_{amp} = kA_p$, where A_p is the area of the pile cross-section and k is given by equation (14). However, it is necessary to ensure that the amplified area do not fall out of the pile cap.

The inclination of the compressive struts is given by $\tan\theta = Z/r$, where $Z = d - 0.5x$. For pile caps on more than two piles, r is the horizontal projection of the strut coming out of a point with coordinates $(0.25a; 0.25b)$ within the column and goes to the farthest pile.

Two-pile caps

Figure 7 shows the relationship between experimental failure loads $P_{u,\exp}$ and theoretical failure loads $P_{u,teo}$ for 37 two-pile caps. As it can be observed, the theoretical failure load predicted by the model is smaller than the experimental failure load for most of the tests. It should be noted that in the theoretical model the partial safety factors for materials were not included, i.e., it was considered $f_{cd} = f_c$ and $f_{yd} = f_y$. Therefore, it can be ensured that the model will provide a conservative design.

Figure 8 shows the histogram of the ratio $R = P_{u,teo}/P_{u,\exp}$ between the theoretical failure load, $P_{u,teo}$, and the experimental failure load, $P_{u,\exp}$.

The mean value of R is $R_m = 0.91$ and the standard deviation is $\sigma_R = 0.08$. The 95th percentile is given by $R_{k,\sup} = R_m + 1.645\sigma_R = 1.04$, which indicates a very low probability of the model to be nonconservative, even without inclusion of the partial safety factors γ_f, γ_c and γ_s. Thus, during the structural design these partial safety factors will cover handily small errors against safety that may occur.

Three-pile caps

Figure 9 shows the relationship between experimental failure loads $P_{u,\exp}$ and theoretical failure loads $P_{u,teo}$ for 21 three-pile caps. As it can be observed, the theoretical failure load predicted by the model is smaller than the experimental failure load for all pile caps tested.

Figure 10 shows the histogram of the ratio $R = P_{u,teo}/P_{u,\exp}$ for the analyzed three-pile caps.

As seen in **Fig. 10**, $R < 1$ has resulted for all 21 pile caps. The 95th percentile is given by $R_{k,\sup} = R_m + 1.645\sigma_R = 0.85$.

Four-pile caps

Figure 11 shows the relationship between experimental failure loads $P_{u,\exp}$ and theoretical failure loads $P_{u,teo}$ for 80 four-pile caps. As it can be observed, the theoretical failure load predicted by the model is smaller than the experimental failure load for most of the tests.

Figure 12 shows the histogram of the ratio $R = P_{u,teo}/P_{u,\exp}$ for the analysed four-pile caps. The 95th percentile is given by $R_{k,\sup} = R_m + 1.645\sigma_R = 1.13$.

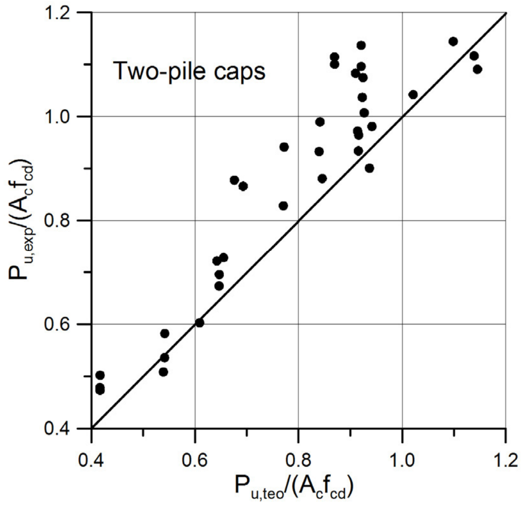


Fig. 7 Relationship between the experimental failure load and the theoretical failure load for two-pile caps.

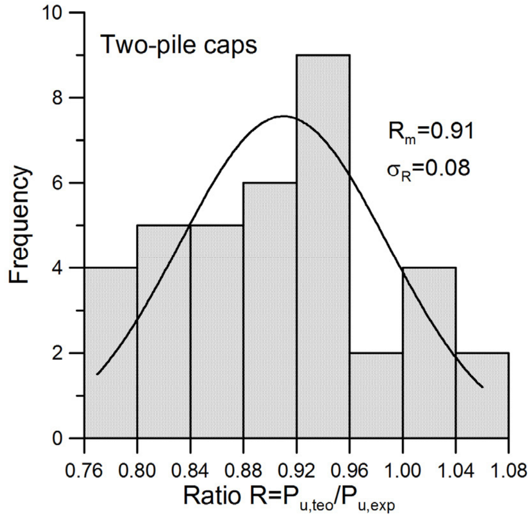


Fig. 8 Histogram for two-pile caps.

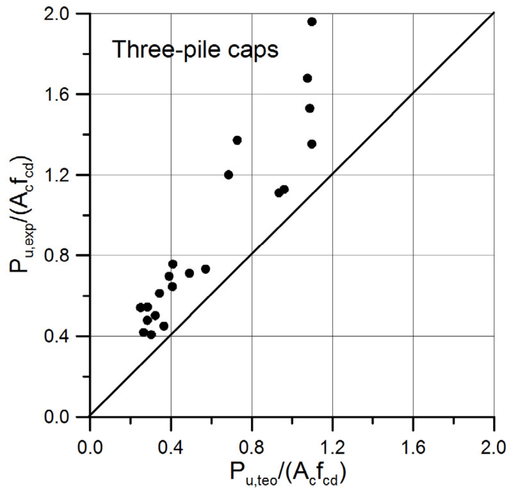


Fig. 9 Relationship between the experimental failure load and the theoretical failure load for three-pile caps.

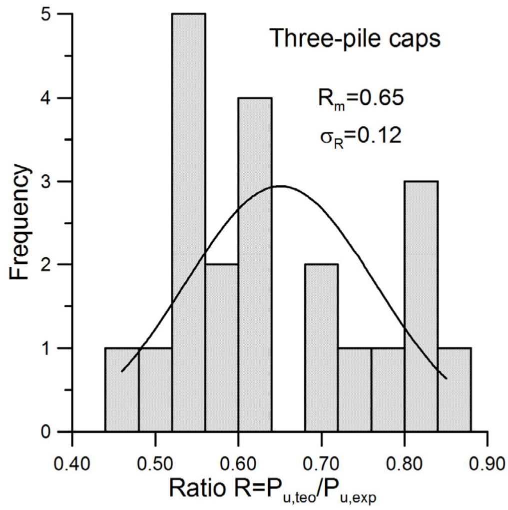


Fig. 10 Histogram for three-pile caps.

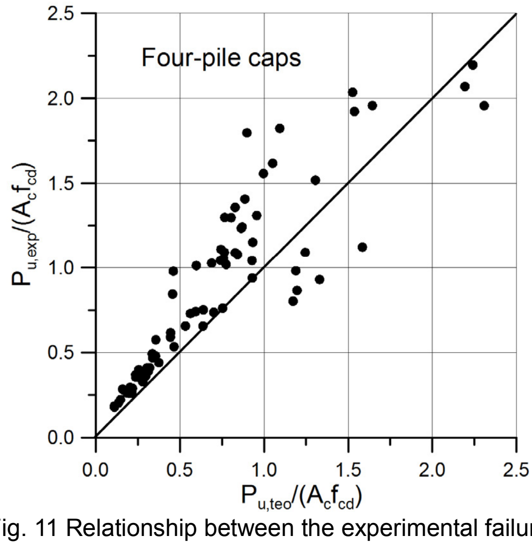


Fig. 11 Relationship between the experimental failure load and the theoretical failure load for four-pile caps.

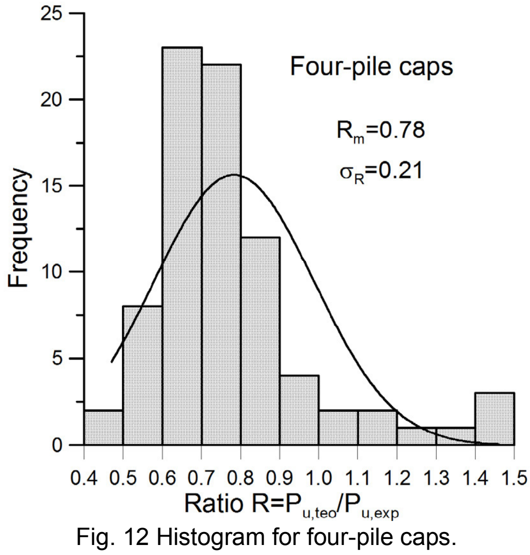


Fig. 12 Histogram for four-pile caps.

Table 2 Failure modes for all pile caps.

	Number of pile caps with:		$R = P_{u,teo} / P_{u,\exp}$	
Number of piles	Concrete crushing	steel yield	R_m	$R_{k,\sup}$
two	26	11	0.91	1.04
three	9	12	0.65	0.85
four	17	63	0.78	1.13

Table 2 indicates the failure modes detected with the proposed model and the mean values R_m and the characteristic values $R_{k,\sup}$ of the relationship $R = P_{u,teo} / P_{u,\exp}$.

4. Numerical examples

When employing the model for the design of pile caps, it is necessary to consider the partial safety factors. The EC2 (2014), for example, adopts $\gamma_c = 1.50$ for concrete and $\gamma_s = 1.15$ for steel for persistent and transient design situations. Moreover, the load factor γ_f depends on the combination of actions considered. For persistent and transient design situations, EN 1990 (2009) adopts the values 1.35 for permanent actions and 1.50 for variable actions.

Example 1:
The proposed model is used to calculate the two-pile cap shown in **Fig. 13**.

Additional data:
$N_d = 1600$ kN; $f_{ck} = 30$ MPa; $f_{yk} = 500$ MPa
Design strengths:
$f_{cd} = 30/1.5 = 20$ MPa (= 2 kN/cm^2)
$f_{yd} = 500/1.15 = 435$ MPa (= 43.5 kN/cm^2)

Relative normal force: $\nu = \dfrac{N_d}{abf_{cd}} = \dfrac{1600}{30x30x2} = 0.89$

$$r = 0.5l_o - 0.25a = 0.5x90 - 0.25x30 = 37.5 \text{ cm}$$

$$\tan\theta_o = \frac{d}{r} = \frac{48}{37.5} \rightarrow \theta_o = 52^o$$

Table 3 shows the results of the iterative process to find x , using the equation (7).
Thus: $x = 15.36$ cm; $x/d = 0.32 < 0.45$; $\theta = 47.07° > 26.6°$
Lever arm: $Z = d - 0.5x = 48 - 0.5x15.36 = 40.32$ cm

Tie steel area: $A_s = \dfrac{0.5x1600x37.5}{40.32x43.5} = 17.10$ cm^2

Strut verification on the pile:

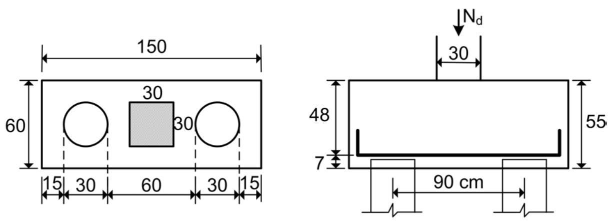


Fig. 13 Example of two-pile cap.

Table 3 Results of the iterative process – Example 1.

Iteration	x (cm)	θ (degrees)	$\dfrac{x_j - x_{j-1}}{x_j}$	$\dfrac{x}{d}$
0	0	52.00	---	0
1	13.17	47.84	1.00	0.27
2	15.00	47.20	0.12	0.31
3	15.31	47.09	0.02	0.32
4	15.36	47.07	<0.01	0.32

$$f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right) f_{cd} = 10.56 \text{ MPa}$$

$F_{de} = 0.5N_d = 800$ kN (pile reaction)
$A_p = 707$ cm^2 (pile cross section area)

$k = 1 + \dfrac{2d'}{\phi_p} = 1 + \dfrac{2x7}{30} = 1.47$ (considering unidirectional spreading)

$A_{amp} = kA_p = 1039$ cm^2 (magnified area on the pile)

$$\sigma_{c2} = \frac{F_{de}}{A_{amp}\sin^2\theta} = \frac{800}{1039\sin^2 47.07} = 1.44 \text{ kN/cm}^2$$

($\sigma_{c2} = 14.4$ MPa)
Since $\sigma_{c2} > f_{cd2}$, it is necessary to consider the bidirectional spreading.

$k = \left(1 + \dfrac{2d'}{\phi_p}\right)^2 = \left(1 + \dfrac{2x7}{30}\right)^2 = 2.15 < 4$ (considering bidirectional spreading)

$A_{amp} = kA_p = 1520$ cm^2 (magnified area on the pile)

$$\sigma_{c2} = \frac{F_{de}}{A_{amp}\sin^2\theta} = \frac{800}{1520\sin^2 47.07} = 0.98 \text{ kN/cm}^2$$

($\sigma_{c2} = 9.8$ MPa)
Since $\sigma_{c2} < f_{cd2}$, the strut safety is ensured.
Transverse reinforcement on the piles:

$$A_{st} = \frac{0.25F_{de}}{f_{yd}} = \frac{0.25x800}{43.5} = 4.60 \text{ cm}^2.$$

Design with the classical method:
Effective concrete strength:
$f_{cd,ef} = 1.2 f_{cd} = 2.40$ kN/cm^2
Verification on the node 1 (below the column):

$$\sigma_{c1} = \frac{N_d}{A_c\sin^2\theta_o} = \frac{1600}{30x30x\sin^2 52} = 2.86 \text{ kN/cm}^2$$

Since $\sigma_{c1} > f_{cd,ef}$, it is necessary to increase the column section according to the classical method.

Example 2:
The same pile cap of the previous example is considered, but subjected to a design load $N_d = 2000$ kN. The relative normal force is $\nu = 1.11$. **Table 4** shows the results of the iterative process in order to find x , using the equation (7).

In this example, the process converges to $x = 25.00$ cm. However, the relation $x/d = 0.52$ is

Table 4 Results of the iterative process – Example 2.

Iteration	x (cm)	θ (degrees)	$\frac{x_j - x_{j-1}}{x_j}$	$\frac{x}{d}$
0	0	52.00	---	0
1	21.18	44.93	1.00	0.44
2	24.21	43.75	0.13	0.50
3	24.85	43.49	0.03	0.52
4	25.00	43.43	<0.01	0.52

greater than 0.45 being expected a brittle failure. The problem may be solved by increasing the height of the pile cap or the column section and repeating the calculations.

Example 3:

The proposed model is used to calculate the four-pile cap shown in **Fig. 14**. The three-dimensional strut-and-tie model is shown in **Fig. 15**.

Additional data:

$d = 58$ cm ; $d' = 7$ cm (total height of the pile cap = 65 cm)

$N_d = 1600$ kN; $M_{xd} = M_{yd} = 160$ kNm

Material properties as in Example 1.

Since the piles have the same axial stiffness, the load on each pile is obtained from the relationship $F_{di} = \frac{N_d}{n} + \frac{M_{xd}}{I_x} x_i + \frac{M_{yd}}{I_y} y_i$, where $n = 4$ is the number of piles, $I_x = \sum_{j=1}^{n} x_j^2$, $I_y = \sum_{j=1}^{n} y_j^2$, x_i and y_i are the coordinates of the pile axis relative to the system of axes $x - y$ passing through the axis of the column.

By substituting the data, results: $F_{d1} = 400$ kN, $F_{d2} = 578$ kN, $F_{d3} = 222$ kN, $F_{d4} = 400$ kN. The length of the horizontal projection of the struts is $r = 53.15$ cm (obtained from the **Fig. 14**).

$$\tan\theta_o = \frac{d}{r} = \frac{58}{53.15} \rightarrow \theta_o = 47.5^o$$

Equivalent load $N_{de} = nF_{d2} = 2312$ kN

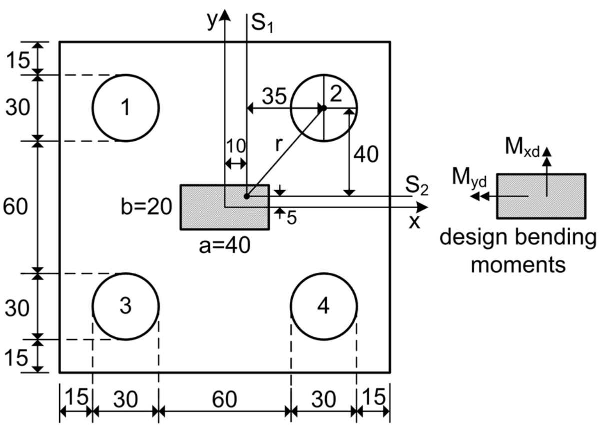


Fig. 14 Example of four-pile cap.

Table 5 Results of the iterative process – Example 3.

Iteration	x (cm)	θ (degrees)	$\frac{x_j - x_{j-1}}{x_j}$	$\frac{x}{d}$
0	0	47.50	---	0
1	9.42	45.08	1.00	0.16
2	9.59	45.03	0.02	0.17
3	9.59	45.03	0.00	0.17

Relative normal force: $\nu = \frac{N_{de}}{abf_{cd}} = \frac{2312}{20x40x2} = 1.45$

Table 5 shows the results of the iterative process to find x, using the equation (10).

Thus: $x = 9.59$ cm; $x/d = 0.17 < 0.45$; $\theta = 45.03° > 26.6°$

Lever arm: $Z = d - 0.5x = 58 - 0.5x9.59 = 53.20$ cm

Tie steel area in the alignment of the piles 1 and 2 (tie T_{12} in **Fig. 15**):

$$A_{sx} = \frac{578x35}{53.20x43.5} = 8.74 \text{ cm}^2$$

Use the same reinforcement for tie T_{34}.

Tie steel area in the alignment of the piles 2 and 4 (tie T_{24} in **Fig. 15**):

$$A_{sy} = \frac{578x40}{53.20x43.5} = 9.99 \text{ cm}^2$$

Use the same reinforcement for tie T_{13}.

Strut verification on the pile 2 (strut C_2 in **Fig. 15**):

$F_{de} = 578$ kN (pile reaction)

$A_p = 707$ cm^2 (pile cross section area)

$$k = \left(1 + \frac{2d'}{\phi_p}\right)^2 = \left(1 + \frac{2x7}{30}\right)^2 = 2.15 < 4$$

$A_{amp} = kA_p = 1520$ cm^2 (magnified area on the pile)

$$\sigma_{c2} = \frac{F_{de}}{A_{amp}\sin^2\theta} = \frac{578}{1520\sin^2 45.03} = 0.76 \text{ kN/cm}^2$$

Since $\sigma_{c2} = 7.6$ MPa is less than $f_{cd2} = 10.56$ MPa, the strut safety is easily ensured.

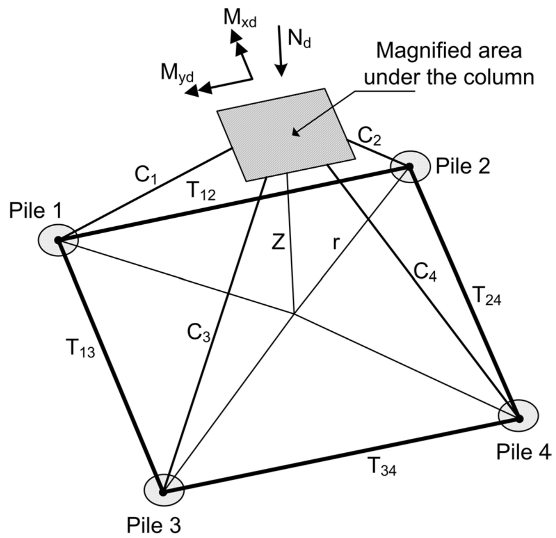


Fig. 15 Three-dimensional strut-and-tie model.

Design with the classical method:
Effective concrete strength:
$f_{cd,ef} = 1.8 f_{cd} = 3.60\,\text{kN/cm}^2$
Verification on the node 1 (below the column):

$$\sigma_{c1} = \frac{N_d}{A_c \sin^2\theta_o} = \frac{1600}{20x40x\sin^2 47.5} = 3.68\,\text{kN/cm}^2$$

Since $\sigma_{c1} > f_{cd,ef}$, it is necessary to increase the column section according to the classical method.

5. Conclusions

A strut-and-tie model for designing concrete pile caps is proposed in this work. The methodology used to evaluate crushing of the concrete struts is the main difference between this model and the classical model proposed by Blévot and Frémy. In their classical model, this check is done for nodes situated directly over the piles and for a node located in the column/pile cap interface. Therefore, they do not consider the amplification of areas over the piles and below the column. This causes a major limitation in the column design.

In the proposed model, crushing of the struts on the piles is verified over a magnified area which considers the propagation of the contact stresses to the reinforcement level. In this verification, a reduced concrete strength is adopted to take into account cracking in the region of tie anchorage. Check of the struts below the column is made on a horizontal plane located into the pile cap. Thus, it is considered that the vertical stress below the column spreads to a depth where it has been reduced enough not to cause crushing of the struts. This verification requires an iterative process. Failure by yielding of the reinforcement is considered in the usual way.

The proposed model was used to analyze 138 pile caps tested by other authors, being 37 two-pile caps, 21 three-pile caps and 80 four-pile caps. The calibration of the model was made using the ratio $R = P_{u,teo}/P_{u,\exp}$ between the theoretical failure load $P_{u,teo}$ and the experimental failure load $P_{u,\exp}$. In the theoretical analysis were not considered partial safety factors. The results showed mean values of R between 0.65 and 0.91, which indicate that the model provides failure loads smaller than those obtained in tests. Thus, when the partial safety factors are considered, a safe design will be obtained.

The proposed model requires an iterative process in order to determine the lever arm and the strut inclination. However, the convergence of the method is very fast and good results are obtained with few iterations, as shown in the numerical examples.

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Notation

A_{amp} = amplified area on the pile
A_c = area of the column cross section
A_s = tie reinforcement area
A_p = area of the pile cross section
F_c = strut compressive force
F_{de} = design pile reaction
M_d = design bending moment
N_d = design load
N_{dc} = design load resisted by the column concrete
N_{de} = design equivalent load
N_{ds} = design load resisted by the column reinforcement
N_k = characteristic axial load
$P_{u,\exp}$ = experimental failure load
$P_{u,teo}$ = theoretical failure load
R = ratio between the theoretical failure load and the experimental failure load
R_m = mean value of the ratio R
R_{sd} = tensile force on the reinforcement
Z = lever arm
a = dimension of the column cross section
b = dimension of the column cross section
d = effective depth of the pile cap
d' = distance between the reinforcement axis and the bottom face of the pile cap
f_c = uniaxial compressive strength of concrete
f_{cd} = uniaxial design compressive strength of concrete
f_{cd1} = concrete compressive strength for CCC nodal zone
f_{cd2} = concrete compressive strength for CCT nodal zone
f_{ck} = characteristic compressive strength of concrete
f_{yd} = design yield strength of reinforcement
f_{yk} = characteristic yield strength of reinforcement
k = amplification factor of area
l_o = distance between the axes of piles
r = length of the horizontal projection of the strut
x = depth from the top of the pile cap
γ_c = partial safety factor for concrete strength
γ_f = partial safety factor for actions
γ_s = partial safety factor for steel strength
θ = strut inclination in the proposed model
θ_o = strut inclination in the classical models
λ = ratio between the sides of the column cross section
ν = relative normal force
σ_c = concrete compressive stress
σ_R = standard deviation of the ratio R
σ_{vd} = vertical compressive stress
ϕ_p = diameter of the pile cross section