
Thermal Cracking of Concrete Pile Caps

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Abstract

The large pile caps of buildings and bridges may have superficial cracks already in the early hours after concreting. Due to the large volume of concrete, the temperature inside the pile cap may reach very high values, as a result of the heat of hydration of cement. Because of the strong temperature gradients, the surface of the pile cap is tensioned and may crack. The employment of skin reinforcement does not avoid the cracking of concrete. However, this reinforcement may reduce the crack width, providing the onset of a large number of small cracks, instead of a single crack with large opening. The object of this work is to address this issue by analyzing the main variables involved and suggest a design methodology for the calculation of skin reinforcement of concrete pile caps.

Keywords: heat transfer, thermal stresses, concrete, cracking, pile caps, reinforcement

1. Introduction

Cement hydration reactions generate heat in concrete and cause a rise in temperature in the core of the structural element, which depends on, among other factors, of size of the element. The higher the structural element, the higher the temperature reached in its core. The heat is transferred by conduction from the core to the surface of the element, where it is dissipated to the environment. Because of this heat transfer, temperature gradients rise, which may introduce tension and compression stresses in concrete. Once the surface cools more quickly, it tends to shorten, while the concrete inside the element is in the heating phase. Thus, the core of the member introduces tensile efforts on superficial layers of concrete. These stresses may cause cracks in the concrete, which may impair its durability.

This is a typical problem of internal or intrinsic imposed deformations and is independent of external loads applied to the structure. Because of the temperature differences between the various points of the structural element, the imposed deformation (thermal deformation) is restrained, which causes compression stresses in the core and tensile stresses on the surfaces of the element. In slender structures, the temperature gradients are small and the tensile stresses are not sufficient to produce cracks in concrete. However, in large elements these surface cracks may be inevitable.

The elements of the framed structure of buildings are usually slender, so there is no need for concern with these cracks. However, pile caps may have sufficiently high dimensions so that this problem is featured in the structural design. In order to minimize this problem, it may be necessary to adopt a set of measures, such as reducing the cement content, using pozzolanic cements, concrete

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placement in layers of small height, precooling and post-cooling of concrete, protection of concrete to avoid very rapid cooling of the surfaces and prolonged cure to reduce shrinkage, among others.

The employment of skin reinforcement does not avoid the cracking of concrete. However, this reinforcement may reduce the crack width, providing the onset of a large number of small cracks, instead of a single crack with large opening. These skin reinforcement mesh to be arranged on all sides of the pile cap.

The problem of concrete cracking resulting from imposed deformations has been extensively studied for one-dimensional elements and for walls [1 to 6]. For three-dimensional structures of large volume, such as dams, there are countless studies on the subject, with approaches aimed at concrete technology and construction techniques, as the concrete placement in layers of small height, precooling and post-cooling the concrete. However, there is a lack of research to quantify the skin reinforcement of pile caps of buildings and bridges. For these structural elements, it is adopted a skin reinforcement from empirical criteria, based on experience, but without any calculation methodology. This is mainly due to the omission of concrete design codes on this topic.

According to the Euro code EC2 [7], for example, the top and the lateral surfaces of pile caps may be unreinforced, provided that there is no risk of cracking of concrete, without presenting any criterion for such check. Other design codes, such as CEB-FIP Model Code [8], ACI 318 [9] and FIB Model Code [10] do not even comment on the subject. As a consequence of this lack of normative guidance, it is usual to find discrepant solutions, from the total absence of skin reinforcement to the employment of visibly excessive skin reinforcement.

The aim of this paper is to study this problem and propose a methodology to calculate the skin reinforcement of large pile caps. The thermal analysis is accomplished with the finite element method [11]. The stresses in concrete are obtained through a simplified analysis of the critical section of the pile cap. The study is limited to two-dimensional heat transfer analysis. However, this analysis may be used with reasonable approximation for pile caps, by defining a width equivalent. This is possible because the heat transfer takes place mainly towards the center to the top of the pile cap.

Due to the presence of the forms on the sides and bottom of the pile cap, which being made of wood provide thermal insulation, the primary heat flow occurs toward the top of the pile cap. For a prismatic pile cap with height H and dimensions A and B in plan, the area of the upper face is AB . A pile cap of same height, but with circular plant of diameter L , has its upper face with area $\pi L^2/4$. To this circular pile cap, the problem is axisymmetric and may be analyzed for a rectangle of width L and height H . Equaling these areas of the two pile caps, it is obtained the equivalent width

$$L = \sqrt{\frac{4AB}{\pi}} \quad (1)$$

2. Analysis of heat transfer

The problem of two-dimensional heat transfer in a material with constant thermal properties, is governed by the differential equation

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} - c\rho \frac{\partial T}{\partial t} + \dot{q}_g = 0 \quad (2)$$

where k_x and k_y are the thermal conductivity in x and y directions, respectively; c is the specific heat; ρ is the density; T is the temperature; \dot{q}_g is the rate of heat generation; t is time. For concrete, it is admitted the isotropy, so that $k = k_x = k_y$.

This differential equation may be solved with use of the finite element method and an integration algorithm step by step. Employing the standard procedure of the finite element method, it is possible to determine temperatures in each point of the concrete pile cap. The formulation of the finite element method (FEM) is presented in detail in references [12, 13]. The boundary conditions of this thermal problem are defined in Fig. 1.

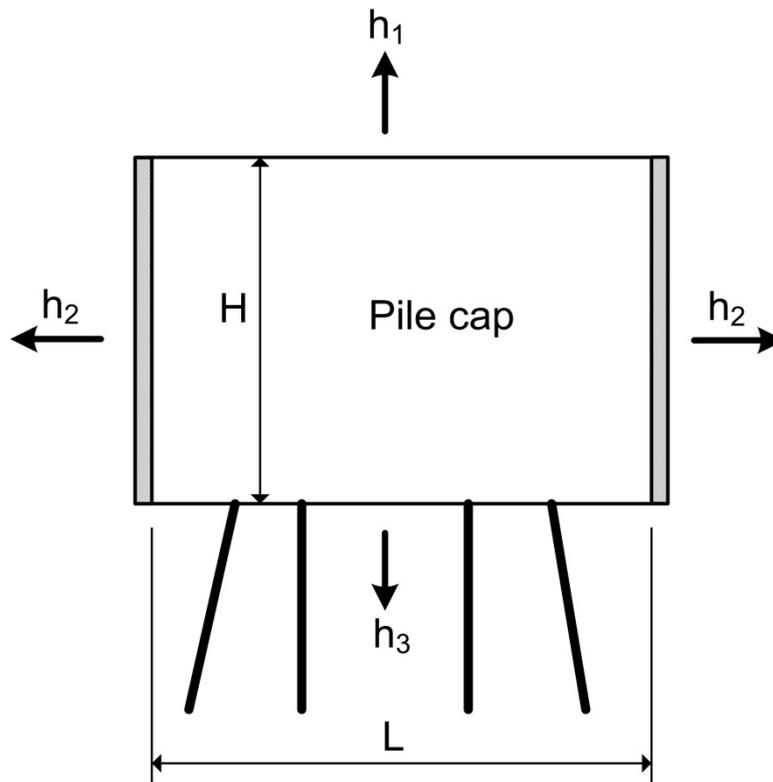


Figure 1. Boundary conditions of the heat transfer problem.

The boundary conditions are specified in all faces of the pile cap using the coefficients of heat transfer by convection h_1 , h_2 and h_3 . These coefficients may vary within a relatively wide range, depending on the wind speed, of the material used as forms and of the thermal resistance of the rock or soil at the bottom of the pile cap. For the top face, it is considered $h_1 = 13.5 \text{ W/m}^2\text{°C}$ as being the mean heat transfer coefficient to air. For the lateral faces, the equivalent transfer coefficient, h_2 takes into account the insulating effect of forms. This coefficient is obtained by means of the equation

$$\frac{1}{h_2} = \frac{1}{h_1} + \frac{t_m}{k_m} \quad (3)$$

where t_m is the thickness of the forms and k_m is the thermal conductivity of its material [14].

Considering, for example, wooden forms with $t_m = 18 \text{ mm}$ and $k_m = 0.14 \text{ W/m}^2\text{°C}$, results $h_2 = 4.93 \text{ W/m}^2\text{°C}$ for the lateral faces (with units $\text{W} = \text{Watt}$, $\text{m} = \text{meter}$ and $\text{°C} = \text{degree Celsius}$). The relationship $\delta = h_2/h_1$ has the value $\delta = 0.365$. After the forms are removed, has $h_2 = 13.5$

W/m² °C for the lateral faces. In this work, it is adopted $h_3 = h_2$ during the entire analysis. Moreover, it is not considered the removal of forms to obtain the greatest temperature gradient toward the top of the pile cap.

3. Thermal properties of concrete

According to Euro code EC2 [7], for temperature of 20°C, the thermal conductivity of concrete varies between $k_{\text{inf}} = 1.33$ W/m°C and $k_{\text{sup}} = 1.95$ W/m°C. The mean value is approximately $k = 1.65$ W/m°C. The specific heat of concrete may be considered equal to $c = 900$ J/kg°C, for temperatures between 20°C and 100°C (with units: J= Joule, kg= kilogram and °C= degree Celsius). The mean density of usual concrete is $\rho = 2400$ kg/m³.

The heat of hydration Q_h is the total amount of heat generated by the complete hydration of the cement. It depends on content and type of cement, as well as the temperature. The rate of hydration also depends on the type of cement. For normal hardening cements, the heat of hydration may reach $Q_h = 350$ kJ/kg at 28 days of age, or a higher value.

The function $T_a(t)$, which represents the adiabatic temperature rise of concrete, varies with the type of cement, the type of aggregate and the water-cement ratio. In this work, it is adopted the expression

$$T_a(t) = T_{a,\text{max}} \left(1 - e^{-0.5t^{0.7}} \right) \quad (4)$$

with the age t in days.

The theoretical curve given by equation (4) is the best fit to a series of concrete mixtures used in Itaipu dam and Tucuruí dam, both located in Brazil [15]. The maximum temperature $T_{a,\text{max}}$ may be obtained from the relationship

$$T_{a,\text{max}} = \frac{Q_{h\infty} M_c}{c \rho} = C_R M_c \quad (5)$$

where $Q_{h\infty}$ is the total hydration heat per kg of cement, M_c represents the cement content per m³ of concrete and $C_R = Q_{h\infty} / (c \rho)$ is the coefficient of thermal efficiency, representing the maximum adiabatic temperature rise per kg of cement per m³ of concrete.

The heat of hydration released until the age t days is given by

$$Q_h(t) = c \rho T_a(t) \quad (6)$$

Finally, considering equations (4), (5) and (6), the rate of heat generation $\dot{q}_g = dQ_h(t)/dt$ may be obtained.

4. Results of thermal analysis

Software based on the finite element method, developed by the Author [12], was employed to analyze pile caps of width L and height H , as shown in Fig. 1. In all the examples, it is considered a concrete hydration heat $Q_{h\infty} = 400$ kJ/kg, which corresponds to the coefficient of

thermal efficiency $C_R = 0.19 \text{ }^\circ\text{C}/(\text{kg}/\text{m}^3)$. The other concrete properties are $k = 1.65 \text{ W}/\text{m}^\circ\text{C}$, $c = 900 \text{ J}/\text{kg}^\circ\text{C}$ and $\rho = 2400 \text{ kg}/\text{m}^3$. The heat transfer coefficients are $h_1 = 13.5 \text{ W}/\text{m}^2\text{C}$ e $h_2 = h_3 = 4.93 \text{ W}/\text{m}^2\text{C}$, with $\delta = h_2/h_1 = 0.365$. It is assumed that the placement temperature of concrete is equal to 25°C and the mean air temperature is equal to 20°C .

Fig. 2 shows the temperature variations at the center, at the top, and the temperature difference between these two locations for a pile cap of width $L = 1.4 \text{ m}$ and height $H = 0.7 \text{ m}$. Cement content is $350 \text{ kg}/\text{m}^3$.

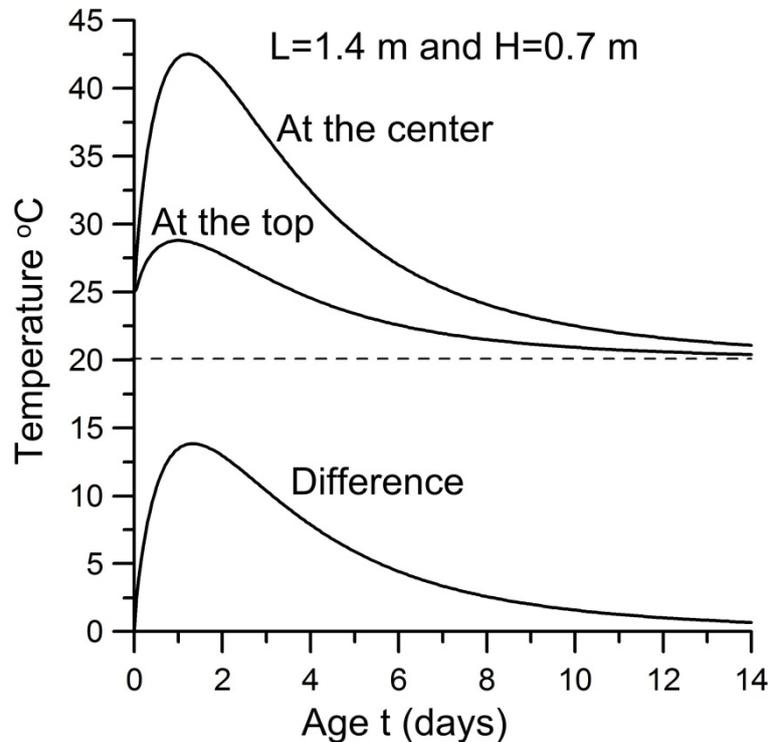


Figure 2. Temperature variations in concrete (pile cap height: $H = 0.7 \text{ m}$).

It is observed that the thermal equilibrium is reached about two weeks after the concrete placement. The peak temperature occurs at an age $t = 1.3 \text{ day}$. The maximum temperature reached inside the pile cap is $T_{\max} = 42.8^\circ\text{C}$. Maximum temperature difference between the core and the top of the pile cap is $\Delta T = 13.8^\circ\text{C}$.

Maximum temperatures depend on the size of the concrete pile cap, in addition to other factors involved in the thermal analysis. For the structural design, it is desirable to find an equation that allows to obtain the maximum temperature difference ΔT between the core and the surface of the pile cap. For this, it is desirable to correlate ΔT with an equivalent thickness of the pile cap.

The equivalent thickness H_e may be defined as being the ratio between the area of the rectangle LH and the perimeter where the heat is lost. In order to take into account the insulating effect of the forms and the thermal resistance imposed by soil, the equivalent thickness is defined as

$$H_e = \frac{LH}{(1 + \delta_3)L + 2\delta_2H} \quad (7)$$

where $\delta_2 = h_2/h_1$ and $\delta_3 = h_3/h_1$ are relationships between the coefficients of heat transfer.

In the numerical examples, it is considered $\delta = \delta_2 = \delta_3$ because $h_2 = h_3 = 4.93 \text{ W/m}^2\text{°C}$. If the insulation is not taken into account, $\delta = 1$ and $H_e = LH/2(L + H)$. If the insulation is complete, $\delta = 0$ and $H_e = H$. In an actual situation, the equivalent thickness varies between these two limits.

In order to determine the correlation between ΔT and H_e , they were analyzed pile caps of width L ranging between 0.3 m to 8.0 m and height H ranging between 0.3 m and 2.0 m. By varying these dimensions, they were obtained various combinations of L/H . In this numerical analysis, it was considered $\delta = 0.365$. Other data remained unchanged, varying only the cement content.

The finite element model, as described in [12, 13], was used to determine the temperature field in the pile cap. Based on the results obtained with this finite element analysis, the following equation was found

$$\Delta T = \frac{(4760 + 90M_c)}{1000} H_e - \frac{(1840 + 9.8M_c)}{1000} H_e^2 \quad (8)$$

where M_c is the cement content in kg/m^3 , H_e is the equivalent thickness in meters and ΔT is given in $^\circ\text{C}$.

Equation (8) was obtained from a two-dimensional thermal analysis. It may be used for three-dimensional pile caps with a proper definition for the width L . For pile caps with circular base, it may be adopted $L = D$, where D is the diameter of the base. For pile caps with a rectangular base, the equivalent width is given in equation (1). If the heat of hydration of cement $Q_{h\infty}$ is different from 400 kJ/kg, it is possible to employ the equation (8) using the equivalent cement content $M_{ce} = (M_c Q_{h\infty})/400$.

The two-dimensional model for heat transfer analysis was compared with a three-dimensional model presented by Klemczac and Knoppik-Wróbel [16]. The results obtained were very similar, which confirms the validity of the two-dimensional analysis, considering the equivalent width given in equation (1).

5. Stress analysis

The determination of stresses in concrete resulting from temperature variations, may be carried out using the finite element method, as described in [12, 13]. In this case, the FEM is used to determine temperature increments and stress increments in each time interval. However, for this particular problem it is possible to make a simplified analysis, considering only the vertical section passing through the center of the pile cap. Since the stresses depend directly on temperature gradients, it is possible to analyze only this vertical section, because it is the one that presents the greatest temperature gradient.

Fig. 3 shows the temperatures and strains on this vertical section.

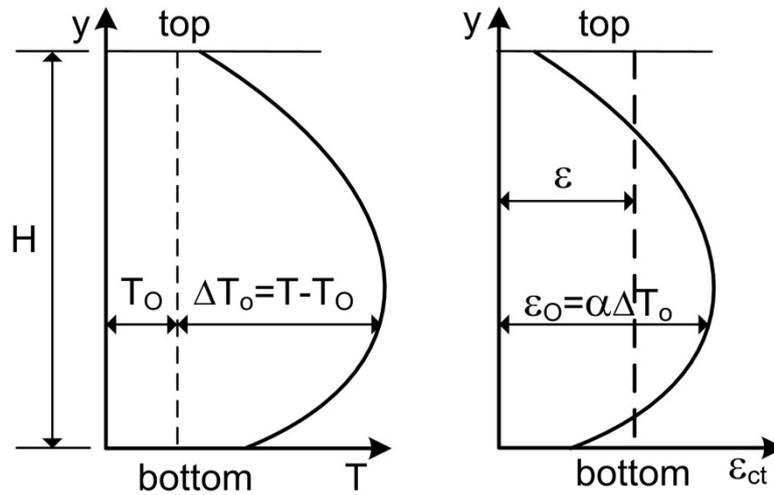


Figure 3. Temperatures and strains in the vertical section.

The variation of temperature in relation to placement temperature T_o , in a point at a distance y from the bottom, is $\Delta T_o = T - T_o$. It is observed that $\Delta T_o = \Delta T_o(t, y)$ is a function of the age t and of the distance y to the bottom.

The free thermal strain in this point of coordinate y is given by

$$\epsilon_o = \alpha \Delta T_o = \alpha(T - T_o) \tag{9}$$

where $\alpha = 10^{-5} \text{C}^{-1}$ is the coefficient of thermal expansion for concrete.

Since the vertical section remains plane and vertical, actual strain ϵ must be constant along the height. Due to this difference between the restrained strain ϵ and the free strain ϵ_o , normal stresses σ_c arise along the height of the section. These stresses depend on the difference $\Delta \epsilon = \epsilon - \epsilon_o$ between the restrained strain and the free strain.

Stresses in concrete are obtained with the stress-strain diagrams shown in Fig. 4. The diagram for tensioned concrete is proposed by CEB-FIP Model Code [8] and also adopted in FIB Model Code [10]. This diagram takes into account the progressive micro-cracking which starts at a stress of about 90% of the tensile strength. The micro-crack grows and forms a discrete crack at stress close to the mean tensile strength f_{ctm} .

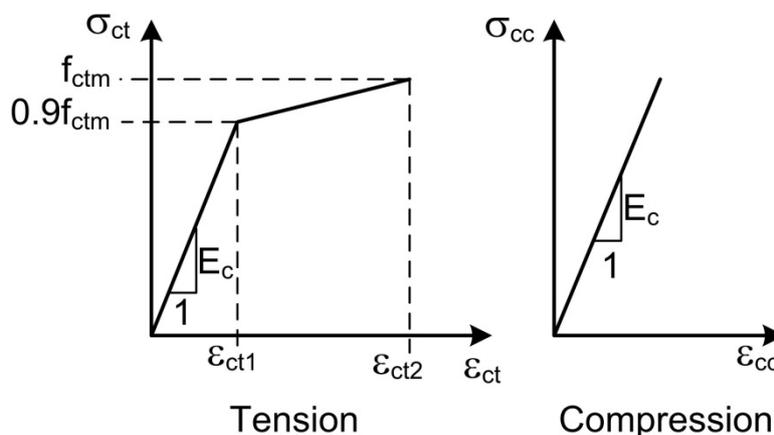


Figure 4. Stress-strain diagrams for plain concrete.

The strain $\varepsilon_{ct1} = 0.9f_{ctm}/E_c$ is evaluated with the mean tensile strength $f_{ctm}(t)$ and the tangent modulus $E_c(t)$ at an age t days. The strain ε_{ct2} is constant and equal to 0.00015. Favorable effects of creep are not considered.

According to CEB-FIP Model Code [8], these properties of concrete at an age t days are given by

$$E_c(t) = [\beta_{cc}(t)]^{1/2} E_{c28} \quad (10)$$

$$f_{ctm}(t) = \beta_{cc}(t) f_{ctm28} \quad (11)$$

where E_{c28} and f_{ctm28} are the tangent modulus and mean tensile strength at the age of 28 days, which may be obtained from the compressive characteristic strength f_{ck} , according to relationships

$$E_{c28} = 21500 \left(\frac{f_{ck} + 8}{10} \right)^{1/3}, \text{ MPa} \quad (12)$$

$$f_{ctm28} = 1,40 \left(\frac{f_{ck}}{10} \right)^{2/3}, \text{ MPa} \quad (13)$$

with f_{ck} given in MPa.

The aging function $\beta_{cc}(t)$ is given by

$$\beta_{cc}(t) = \exp \left\{ s \left[1 - \left(\frac{28}{t} \right)^{1/2} \right] \right\} \quad (14)$$

where s takes into account the type of cement (CEB 1993)

In order to perform an early-age structural analysis, it is necessary to establish a minimum age from which the mechanical properties of concrete are measurable. In general, it may be assumed that age as 12 hours, which corresponds to a degree of hydration of about 20% for the normal hardening cements [17]. Therefore, the stress analysis is performed only for $t > 0.5$ day.

If the restrained strain ε is known along the height of the pile cap, the difference $\Delta\varepsilon = \varepsilon - \alpha(T - T_o)$ may be evaluated in each point situated at a distance y from the bottom. Using the stress-strain diagrams of concrete, one obtains the stress $\sigma_c = \sigma_c(t, y)$. As the normal effort in this central section is null, one should have

$$\int_0^H \sigma_c dy = 0 \quad (15)$$

Equation (15) allows us to determine the restrained strain ε through an iterative process. A numeric integration is performed in each iteration. Then, it is estimated $\Delta\varepsilon = \varepsilon - \alpha(T - T_o)$, yielding the stresses $\sigma_c = \sigma_c(t, y)$ along the height of the pile cap, as illustrated in Fig. 5.

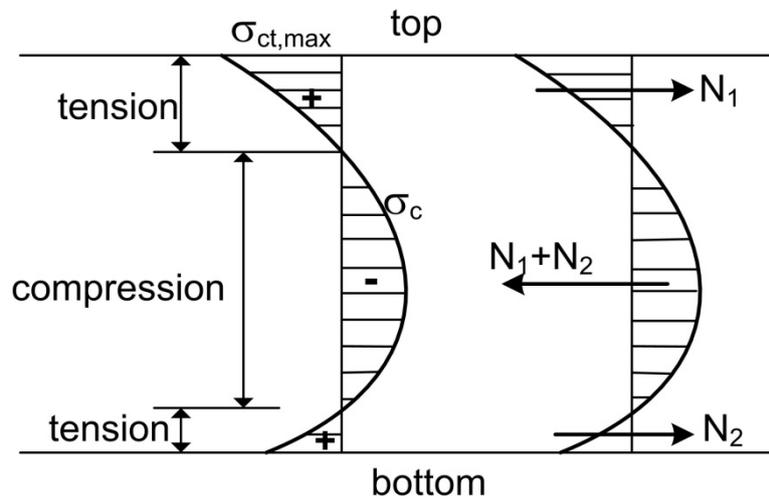


Figure 5. Normal stresses in the vertical section.

The layers near the top and bottom of the pile cap are under tension, while the central region is compressed. Tensile normal efforts along these faces are N_1 and N_2 , and may be obtained by numerical integration. The maximum tensile stress $\sigma_{ct,max}$ occurs at the top of the pile cap. Cracking occurs when $\sigma_{ct,max} = f_{ctm}$, where $f_{ctm} = f_{ctm}(t_r)$ is the mean tensile strength of concrete in the age t_r days. At this moment, the effort N_1 is equal to the cracking normal effort N_r . This cracking effort may be written as

$$N_r = h_o f_{ctm28} \tag{16}$$

where f_{ctm28} is the mean tensile strength of concrete at the age of 28 days and h_o is the thickness of the surface layer that is relevant to calculate the minimum reinforcement.

6. Results of stress analysis

Results presented below were obtained for a set of pile caps with width varying from $L = 0.3$ m to $L = 8.0$ m and height ranging from $H = 0.3$ m to $H = 2.0$ m. In total they were made 24 combinations of these two dimensions. Moreover, the cement content varied between $M_c = 300$ kg/m³ and $M_c = 400$ kg/m³. For the compressive strength of concrete they were considered values of 20 MPa, 25 MPa, 30 MPa and 40 MPa. It is assumed normal hardening cement, with $s = 0.25$ in equation (14). Data for thermal analysis are the same as those adopted previously.

Fig. 6 shows the relationship between the thickness of the surface layer h_o , as defined in equation (16), and the equivalent thickness H_e . As observed, the values of h_o depend on the cement content. However, there is no a correlation with the compressive strength of concrete or with the equivalent thickness.

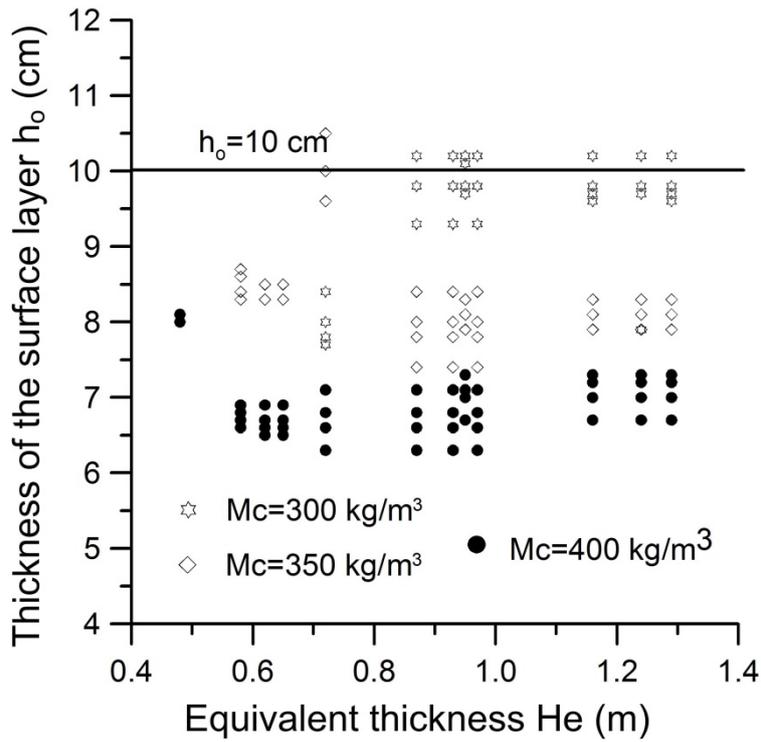


Figure 6. Thickness of the surface layer to calculate the minimum reinforcement.

Fig. 7 shows variations of the thickness h_o with the maximum adiabatic temperature $T_{a,max}$, as defined in equation (4).

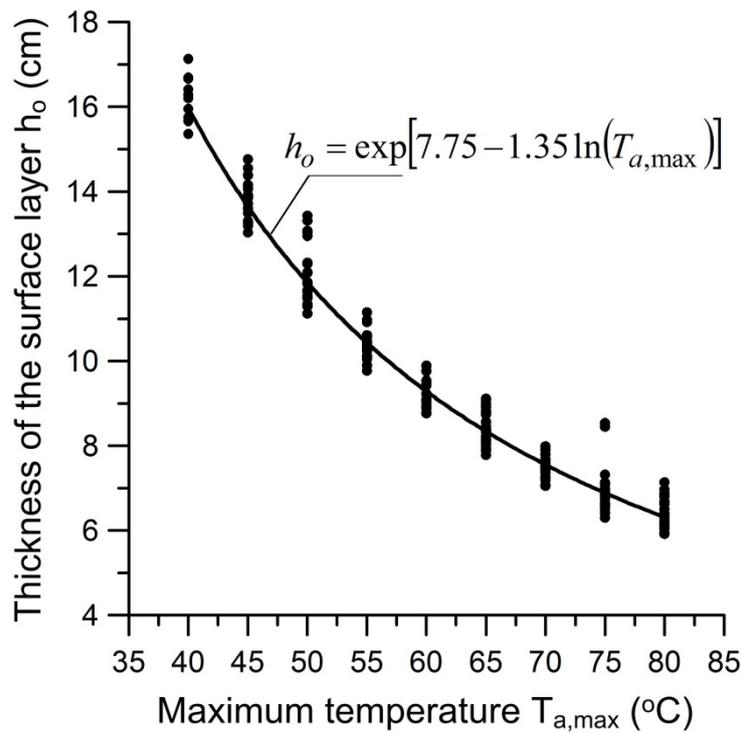


Figure 7. Variation of h_o with $T_{a,max}$ ($R^2=0.98$).

As shown in Fig. 7, there is a direct correlation between the thickness of the surface layer h_o and the maximum adiabatic temperature rise. The adjustment equation is shown in Fig. 7. The

coefficient of determination is $R^2=0.98$. Anyway, it is recommended to consider a minimum value of 10 cm for h_o , as a prudent criterion.

In order to calculate the crack width, it is convenient to correlate the maximum tensile stress in concrete with the maximum temperature difference ΔT between the core and the top of the pile cap. As seen, this temperature difference may be correlated with the equivalent thickness H_e and the cement content M_c , by means of the equation (8). The maximum stress $\sigma_{ct,max}$ may be written as

$$\sigma_{ct,max} = R E_c(t) \alpha \Delta T \tag{17}$$

where R is the restraint factor for imposed deformations.

At the moment of cracking, $t = t_r$, $\sigma_{ct,max} = f_{ctm}(t_r)$ and $\Delta T = \Delta T_{cr}$. Thus, equation (17) may be written as

$$R = \frac{f_{ctm}(t_r)}{E_c(t_r) \alpha \Delta T_{cr}} \tag{18}$$

this makes it possible to determine the restraint factor.

Values of R obtained through the equation (18) are shown in Fig. 8. It is observed that there is no direct correlation between R and the equivalent thickness H_e . The maximum value obtained was equal to 0.32. Assuming a safety factor equal to 1.4, its design value is $R = 1.4 \times 0.32 = 0.45$. Soon, all cases analyzed are handily covered by the usual value $R = 0.5$.

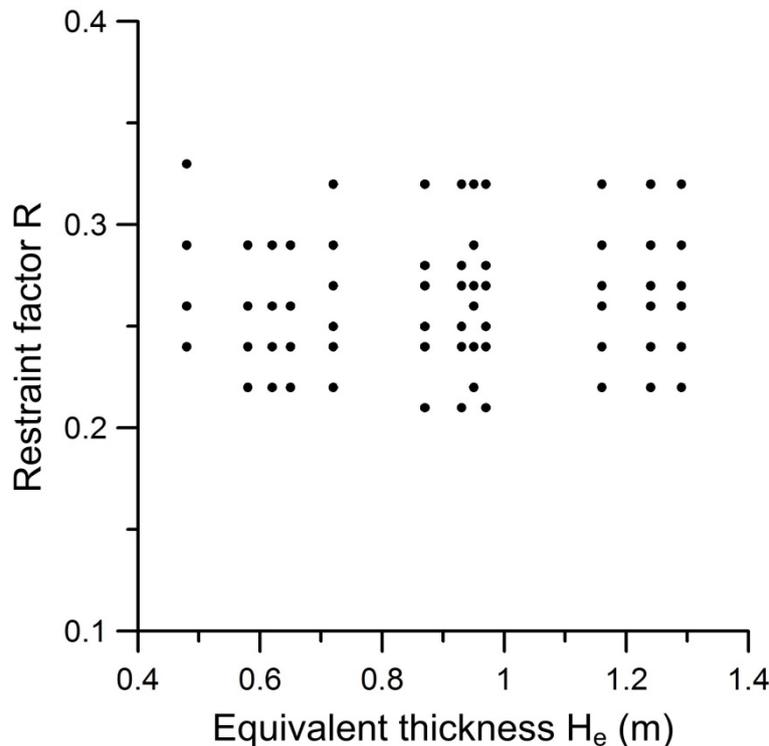


Figure 8. Restraint factors obtained with the FEM.

Fig. 9 shows the variation of ΔT_{cr} with respect to the equivalent thickness H_e . The points indicated in Figure 9 show a reasonable dispersion about the line of least squares (coefficient

of determination $R^2=0.42$). However, the maximum difference between the temperature obtained with the FEM and the one that is predicted with the adjustment equation is lower than $1.2\text{ }^\circ\text{C}$.

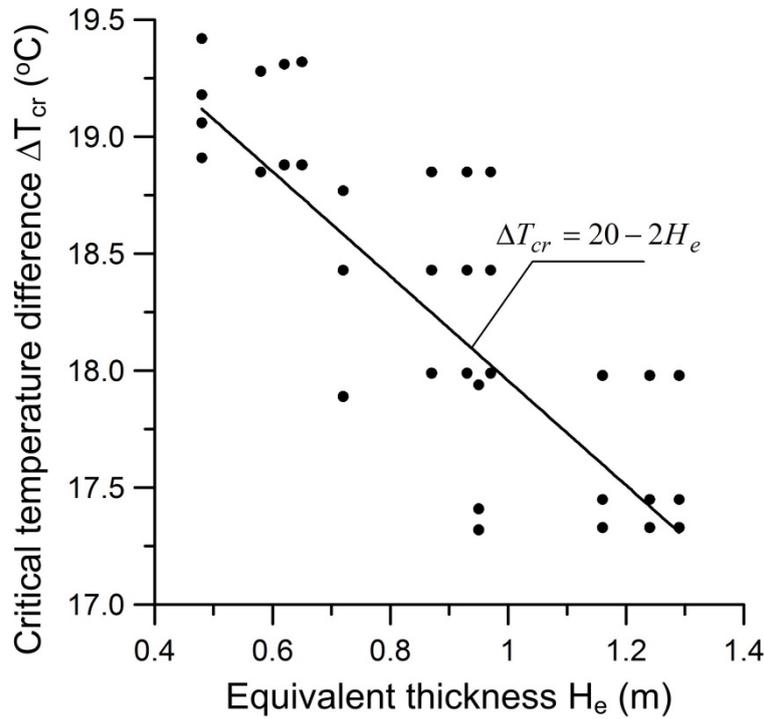


Figure 9. Critical temperature difference at the moment of cracking.

7. Design procedure

As a result of this study, the following procedure may be recommended for design of skin reinforcement of concrete pile caps:

Data:

Pile cap dimensions: A and B in plan and height H , in meters.

Concrete: characteristic strength f_{ck} (MPa) and cement content M_c (kg/m^3).

Checking the risk of thermal cracking:

Equivalent width (m): $L = \sqrt{\frac{4AB}{\pi}}$ (see equation (1))

Equivalent thickness (m): $H_e = \frac{LH}{(1 + \delta)L + 2\delta H}$, with $\delta = 0.365$ (see equation (7))

Maximum temperature difference between the center and the top of the pile cap ($^\circ\text{C}$):

$$\Delta T = \frac{(4760 + 90M_c)}{1000} H_e - \frac{(1840 + 9.8M_c)}{1000} H_e^2 \quad (\text{see equation (8)})$$

If the heat of hydration of cement $Q_{h\infty}$ is different from 400 kJ/kg, it may be used the equivalent cement content $M_{ce} = (M_c Q_{h\infty})/400$ in place of M_c .

Critical temperature difference ($^{\circ}\text{C}$): $\Delta T_{cr} = 20 - 2H_e$ (see Fig. 9)

- If $\Delta T < \Delta T_{cr}$, there is no risk of thermal cracking.

In order to avoid cracking caused by thermal shock and/or differential shrinkage, it may be adopted a skin reinforcement of approximately $2\text{cm}^2/\text{m}$ on the pile cap sides. This is the skin reinforcement for deep beams, as recommended by ACI 318 [9].

- If $\Delta T > \Delta T_{cr}$, there is risk of thermal cracking of the pile caps sides. In this case, it is necessary to calculate skin reinforcement to control the crack width.

Calculation of the skin reinforcement:

A) *Minimum reinforcement to support the cracking effort:*

Adiabatic temperature rise: $T_{a,\max} = \frac{Q_{h\infty} M_c}{c \rho}$, (see equation (5)),

where $Q_{h\infty}$ is the heat of hydration per kg of cement (in kJ/kg), M_c represents the cement content per m^3 of concrete (in kg/m^3), $c = 0.9 \text{ kJ}/\text{kg}^{\circ}\text{C}$ is the specific heat of concrete and $\rho = 2400 \text{ kg}/\text{m}^3$ is the density of concrete.

Thickness of the surface layer: $h_o = \exp[7.75 - 1.35 \ln(T_{a,\max})] \geq 10 \text{ cm}$ (see Fig. 7)

Minimum reinforcement area: $A_{s,\min} = \frac{100 h_o f_{cm28}}{f_{yd}}$, cm^2/m

where f_{cm28} is the mean tensile strength of concrete at the age of 28 days and f_{yd} is the design yield strength of steel.

B) *Reinforcement required for crack width control:*

According to the cracking model of CEB-FIP Model Code [8], the reinforcement ratio for crack width control is given by

$$\rho_{se} = \frac{\phi R \varepsilon_{cn}}{3.6 w_{k,\lim}}$$

where:

ϕ is the diameter of the steel bar, in mm;

$\varepsilon_{cn} = \alpha \Delta T$ is the imposed deformation, with $\alpha = 10^{-5} \text{ }^{\circ}\text{C}^{-1}$ (coefficient of thermal expansion of concrete);

$R = 0.5$ is the restraint factor;

$w_{k,\lim}$ is the limit value of crack width, in mm.

The reinforcement area is $A_s = \rho_{se} h_e$, where h_e is the thickness that is relevant for calculation of crack width, given by $h_e = 2,5(c + 0,5\phi)$, where c is the nominal concrete cover of the steel bars.

If to result $A_s < A_{s,\min}$, it is adopted $A_s = A_{s,\min}$.

8. Numerical examples

In the two examples below, we consider the following properties of concrete:

Characteristic strength: $f_{ck} = 25$ MPa

Heat of hydration: $Q_{h\infty} = 400$ kJ/kg

Specific heat: $c = 0.9$ kJ/kg°C

Density: $\rho = 2400$ kg/m³

Estimated cement content: $M_c = 380$ kg/m³

Since $Q_{h\infty} = 400$ kJ/kg, it is considered the actual cement content.

Example 1:

Pile cap dimensions: $A = 1.60$ m, $B = 1.60$ m, $H = 0.70$ m

Calculations:

$$\text{Equivalent width: } L = \sqrt{\frac{4AB}{\pi}} = \sqrt{\frac{4 \times 1.60 \times 1.60}{\pi}} \rightarrow L = 1.81 \text{ m}$$

$$\text{Equivalent thickness: } H_e = \frac{LH}{(1 + \delta)L + 2\delta H} = \frac{1.81 \times 0.70}{(1 + 0.365)1.81 + 2 \times 0.365 \times 0.70} \rightarrow H_e = 0.42 \text{ m}$$

$$\text{Maximum temperature difference: } \Delta T = \frac{(4760 + 90M_c)}{1000} H_e - \frac{(1840 + 9.8M_c)}{1000} H_e^2$$

$$\Delta T = \frac{(4760 + 90 \times 380)}{1000} 0,42 - \frac{(1840 + 9.8 \times 380)}{1000} 0,42^2 \rightarrow \Delta T = 15.4 \text{ }^\circ\text{C}$$

$$\text{Critical temperature difference: } \Delta T_{cr} = 20 - 2H_e = 20 - 2 \times 0.42 \rightarrow \Delta T_{cr} = 19.2 \text{ }^\circ\text{C}$$

Since $\Delta T < \Delta T_{cr}$, there is no risk of thermal cracking.

In order to avoid cracking caused by thermal shock and/or differential shrinkage, it may be adopted a skin reinforcement of 2cm²/m on the pile cap sides.

Example 2:

Pile cap dimensions: $A = 4.00$ m, $B = 4.00$ m, $H = 1.60$ m

Calculations:

$$\text{Equivalent width: } L = \sqrt{\frac{4AB}{\pi}} = \sqrt{\frac{4 \times 4.00 \times 4.00}{\pi}} \rightarrow L = 4.51 \text{ m}$$

$$\text{Equivalent thickness: } H_e = \frac{LH}{(1 + \delta)L + 2\delta H} = \frac{4.51 \times 1.60}{(1 + 0.365)4.51 + 2 \times 0.365 \times 1.60} \rightarrow H_e = 0.99 \text{ m}$$

$$\text{Maximum temperature difference: } \Delta T = \frac{(4760 + 90M_c)}{1000} H_e - \frac{(1840 + 9.8M_c)}{1000} H_e^2$$

$$\Delta T = \frac{(4760 + 90 \times 380)}{1000} 0,99 - \frac{(1840 + 9.8 \times 380)}{1000} 0,99^2 \rightarrow \Delta T = 33.1^\circ\text{C}$$

$$\text{Critical temperature difference: } \Delta T_{cr} = 20 - 2H_e = 20 - 2 \times 0,99 \rightarrow \Delta T_{cr} = 18.0^\circ\text{C}$$

Since $\Delta T > \Delta T_{cr}$, it is necessary to check the thermal cracking.

A) Minimum reinforcement to support the cracking effort:

$$T_{a,\max} = \frac{Q_{h\infty} M_c}{c \rho} = \frac{400 \times 380}{0.9 \times 2400} = 70.4^\circ\text{C}$$

$$h_o = \exp[7.75 - 1.35 \ln(T_{a,\max})] = \exp[7.75 - 1.35 \ln(70.4)] \rightarrow h_o = 7.4 \text{ cm}$$

As it resulted $h_o < 10 \text{ cm}$, it is adopted the minimum thickness: $h_o = 10 \text{ cm}$.

$$f_{ctm28} = 1,40 \left(\frac{f_{ck}}{10} \right)^{2/3}, \text{ MPa (see equation (13))}$$

$$f_{ctm28} = 1,40 \left(\frac{25}{10} \right)^{2/3} \rightarrow f_{ctm28} = 2.58 \text{ MPa}$$

$$f_{yd} = 435 \text{ MPa (design yield strength of steel)}$$

$$A_{s,\min} = \frac{100 h_o f_{ctm28}}{f_{yd}} = \frac{100 \times 10 \times 2.58}{435} \rightarrow A_{s,\min} = 5.93 \text{ cm}^2/\text{m}$$

B) Reinforcement required for crack width control:

$$\text{Imposed deformation: } \varepsilon_{cn} = \alpha \Delta T = 10^{-5} \times 33.1 \rightarrow \varepsilon_{cn} = 33.1 \times 10^{-5} \text{ (imposed deformation)}$$

$$\text{Limit value of crack width: } w_{k,\lim} = 0.2 \text{ mm (adopted as a criterion of durability)}$$

$$\text{Diameter of the steel bar: } \phi = 10 \text{ mm (adopted)}$$

$$\text{Nominal concrete cover: } c = 5 \text{ cm (adopted as a criterion of durability)}$$

$$\rho_{se} = \frac{\phi R \varepsilon_{cn}}{3.6 w_{k,\lim}} = \frac{10 \times 0.5 (33.1 \times 10^{-5})}{3.6 \times 0.2} \rightarrow \rho_{se} = 0.23\%$$

$$h_e = 2,5(c + 0,5\phi) = 2,5(5 + 0.5 \times 1) \rightarrow h_e = 13.75 \text{ cm}$$

$$A_s = \rho_{se} 100h_e = \frac{0.23}{100} \times 100 \times 13.75 = 3.16 \text{ cm}^2/\text{m}$$

As it resulted in $A_s < A_{s,\min}$, it is adopted $A_s = 5.93 \text{ cm}^2/\text{m}$.

Solution: bars of $\phi = 10 \text{ mm}$ spaced at 13 cm.

9. Conclusions

Software based on the finite element method (FEM) and developed by the Author [12], was employed for two-dimensional analysis of heat transfer. Through the definition of a width equivalent, this method was used to determine temperature rise in concrete pile caps due to heat of hydration of cement. The temperature distribution obtained with FEM was used to perform the stress analysis of the critical section of pile cap. The main parameters involved were correlated with an equivalent thickness of the pile cap.

Based on the obtained results, it was proposed a methodology to calculate the skin reinforcement of large concrete pile caps. This reinforcement is intended to control the crack width resulting from thermal stresses due to hydration of cement. It should be noted that the skin reinforcement does not prevent cracking. It only limits the crack width, avoiding the emergence of a large crack that might compromise the durability of the structural element. For massive pile caps, it is necessary to adopt a set of measures, such as reducing the cement content, using pozzolanic cements, concrete placement in layers of small height, precooling and post-cooling of concrete, protection of concrete to avoid very rapid cooling of the surfaces, prolonged cure to reduce shrinkage, among others. The employment of skin reinforcement, solely, does not eliminate the need for such additional cautions.

Results of thermal analysis are very dependent on the concrete properties and boundary conditions. So it should be clear that the results obtained, as well as the proposed design methodology, are limited to the conditions that have been adopted in this work. The method for checking the risk of thermal cracking and calculating skin reinforcement may be used to pile caps of equivalent height up to 2.0 m. For larger pile caps, it is necessary to perform specific study, considering thermal properties and boundary conditions corresponding to the actual situation.

References

- [1] BS 8007:1987; *Code of practice for Design of concrete structures for retaining aqueous liquids*. London, 1987.
- [2] Harrison T.A.; *Early-age Thermal Crack Control in Concrete*, CIRIA Report 91, London, 1992.
- [3] EN 1992-3:2006; *Eurocode 2: Design of Concrete Structures – Part 3: Liquid retaining and containment structures*. Brussels, 2006.
- [4] Bamforth P.; *Early-Age Thermal Crack Control in Concrete*, CIRIA Report C660:2007, London, 2007.
- [5] Flaga K., Furtak K.; *Problem of thermal and shrinkage cracking in tanks vertical walls and retaining walls near their contact with solid foundation slabs*, Architecture Civil Engineering Environment, ACEE, 2 (2), p.23-30, 2009.
- [6] Bamforth P., Denton S., Shave J.; *The development of a revised unified approach for the design of reinforcement to control cracking in concrete resulting from restrained contraction*, Report ICE/0706/012, London, 2010.

- [7] EN 1992-1-1:2010; *Eurocode 2: Design of concrete structures – Parte 1-1: General rules and rules for buildings*. Brussels, 2010.
- [8] *CEB-FIP Model Code 1990*. Published by Thomas Telford, London, 1993.
- [9] ACI 318M-11:2011; *Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary*. U.S.A., ACI, 2011.
- [10] *FIB Model Code 2010*. Lausanne, 2010.
- [11] Zienkiewicz O.C., Taylor R.L.; *The Finite Element Method*. 7nd ed. UK: Butterworth-Heinemann, 2013.
- [12] Araújo, J. M. *Analysis of concrete gravity dams considering the construction phase and the dynamic interaction dam-reservoir-foundation*. Doctoral thesis, Graduate Program in Civil Engineering, Federal University of Rio Grande do Sul, Porto Alegre, Brazil, 1995. (in Portuguese).
- [13] Araújo, J. M.; Awruch, A. M. Cracking safety evaluation on gravity concrete dams during the construction phase. *Computers and Structures*, 66 (1), pp.93-104, 1998.
- [14] Serth R.W.; *Process Heat Transfer. Principles and Applications*. U.S.A.: Academic Press, 2007.
- [15] Andrade W.P., Fontoura J.T.F., Bittencourt R.M., Guerra E.A.; *Adiabatic temperature rise of concrete, Bulletin IBRACON M-4*, São Paulo: IBRACON, 1981, (in Portuguese).
- [16] Klemczac B., Knoppik-Wróbel A.; *Early age thermal and shrinkage cracks in concrete structures - influence of geometry and dimensions of a structure*, *Architecture Civil Engineering Environment, ACEE*, 4 (3), pp. 55-70, 2011.
- [17] Atrushi D.S.; *Tensile and Compressive Creep of Early Age Concrete: Testing and Modelling*. Doctoral Thesis, Department of Civil Engineering, The Norwegian University of Science and Technology, Trondheim, Norway, 2003.