Artigo publicado na revista Advances in Engineering Software, Vol.32, No.12, p.871-879, 2001.

PROBABILISTIC ANALYSIS OF REINFORCED CONCRETE COLUMNS

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ABSTRACT

The subject of this work is the probabilistic finite element analysis of reinforced concrete columns. Concrete properties are represented as homogeneous Gaussian random fields. The yield stress and position of steel reinforcement, dimensions of the column cross section and axial load are considered as random variables. The Monte Carlo method is employed to obtain expected values and standard deviations of the rupture load. The partial safety factors method is used for columns design and structural safety is evaluated by means of the reliability index, which is obtained through simulations. The effects of main parameters on the reliability index are investigated. It is shown that the correlation length of random fields for concrete properties may have a significant effect on reliability. Therefore, simplified procedures, which do not consider spatial variations of concrete properties are inappropriate for safety analysis.

Keywords: Probabilistic analysis, structural safety, Monte Carlo simulation, reinforced concrete, finite element method

1 INTRODUCTION

The analysis of a reinforced concrete structure involves several uncertainties related to concrete and steel properties, structural dimensions and position of steel reinforcement, loads and boundary conditions. Furthermore, uncertainties associated to the employed constitutive models are unavoidable. Therefore, for a realistic analysis, it is necessary to look for expected values and variances of the structural response, considering random input parameters.

Several methods for probabilistic structural analysis have been studied in the last years, particularly in association with the finite element method. Generally, the methods employed are the Monte Carlo method, the Monte Carlo simulation with Neumann expansion and perturbation techniques [1-4].

The Monte Carlo method is the most simple and evident way to accomplish a probabilistic analysis, and for that reason is widely used. In this method, material properties, loads and other random variables are introduced by digital simulation, without any significant modification of the algorithm used in the deterministic analysis. Moreover, Monte Carlo method is statistically consistent and may be employed to test other techniques. However, Monte Carlo method may be computationally very expensive in problems with several degrees of freedom and when many simulations are necessary to obtain the statistical descriptors of the structural response.

In the design criteria of reinforced concrete structures based on ultimate limit states, safety is reached by means of partial safety factors. These factors are introduced with different values to increase or to reduce the magnitude of the random variables involved in the analysis [5-7].

Usually, loads, materials strengths and structural dimensions are the basic random variables considered in the design. Partial safety factors are introduced to increase loads and to reduce steel and concrete strengths. For concrete and steel, the partial factors cover the deviations of the nominal dimensions and the difference between the strength obtained from test specimens and the strength in the actual structure [6].

The use of partial safety factors, although convenient, is not sufficient to determine the safety level obtained in the design. In fact, safety depends on the structural response due to the actions and this involves interdependence among all random variables.

A consistent evaluation of the safety level requires the determination of the structural failure probability. This probability can be estimated if the probability distribution of a certain random variable representing a given safety margin for the structure is known. Unfortunately, it is not always possible or practical to obtain this probability distribution.

An alternative to obtain the safety level consists in the evaluation of the reliability index [8]. This index, takes into account all random variables involved and the way the structure responds to the actions. The reliability index is associated to a failure probability, although this relationship is not explicit.

Several works have been developed to evaluate the reliability of reinforced concrete columns [9-12]. The main objective of these works is to determine the level of reliability obtained with design codes to establish numerical values for safety factors, which lead to a desirable reliability index. Structural analysis is accomplished by assuming a simplified shape for the deformed axis of the column and considering concrete properties as random variables.

These procedures have some important drawbacks. For a symmetrical loading the assumed shape for the deformed axis of the column is also symmetrical as expected in a deterministic analysis. This simplification is adopted because concrete properties are considered as random variables (without spatial variations along the column length). Such a simplification may introduce a strong deterministic component: the predetermination of the cross section where failure occurs.

When concrete properties are considered as random fields (with spatial variations) it is verified that failure may begin in any cross section of the column. Besides, symmetrical loading causes asymmetric deflection depending of the random properties distribution on the column length. Thus, it is necessary to consider concrete properties as random fields.

Random spatial variations of the material properties may have a significant effect on reliability depending on correlation lengths of the random fields [13]. As will be shown, for a reinforced concrete column the reliability index sharply decreases as the correlation length increases. Therefore, concrete properties can't be modeled as random variables.

In a probabilistic finite element analysis, random fields may introduce a strong mesh-dependency. This mesh-dependency is due to random spatial fluctuations in the material properties, which are related to the correlation lengths. Thus, the mesh selection is an important task in the finite element reliability analysis.

In this work, the Monte Carlo method is employed to evaluate the reliability index obtained with the usual design procedures of reinforced concrete columns. Columns are parts of non-sway framed structures and are bent in single curvature under short-time loads. Effects of the main parameters on the reliability index are investigated. The finite element method is used in the structural analysis, including material and geometrical nonlinearities. Some available experimental results are used to validate the adopted model. Load-path-dependency and load-correlation effects are not considered.

2 THE STOCHASTIC FIELD GENERATION

When employing the Monte Carlo method it is necessary to generate several samples of the considered random variables. For each sample, the structure is analyzed and several values of the response are obtained. Thus, the first step to be taken is to define the basic random variables.

The basic random variables considered in this work are:

E_c, f_c, f_t = modulus of elasticity, compressive strength and tensile strength of the concrete;

f_y = yield stress of the steel reinforcement;

b, h = width and height of the column cross section (rectangular sections);

η = distance of steel bars to the center of gravity of the concrete section;

F = compressive axial load.

The steel modulus of elasticity, the initial eccentricity of the axial load, the length of the column and boundary conditions are considered as deterministic.

Concrete properties present random variations over the structural domain defined by autocorrelation functions. Other random variables are considered constant with values obtained in each simulation, i.e. without spatial variations. Material properties are considered as Gaussian random variables. Cross section dimensions and positions of the steel bars present uniform random variations around the mean values.

A value for the cross section width, b_i, corresponding to simulation of order i, is given by

$$b_i = b_m\left[1 + c\left(1 - 2S_i\right)\right] \tag{1}$$

where b_m represents the mean width and S_i is a random number with uniform distribution in the interval $[0,1]$.

Constant c is taken as

$$c = \begin{cases} 0.025 \ , & \text{if } \ b_m \leq 200 \\ 0.008 + 3.4/b_m \ , & \text{if } \ 200 < b_m \leq 2000 \\ 0.001 + 17.4/b_m \ , & \text{if } \ b_m > 2000 \end{cases} \tag{2}$$

with b_m given in mm.

Employing equation (1) it is possible to generate several values for the cross section width. A similar expression is used to generate cross section height in each simulation.

Variable η, which defines the reinforcement locations with respect to the section center of gravity, is given by

$$\eta_i = \eta_m \left[1 + 0.01 \left(1 - 2S_i \right) \right] \tag{3}$$

where η_m is the mean value of η.

Equations (1) and (3) agree with the dimensional variations admitted in the CEB-FIP Model Code/90[6]. These equations are employed to generate the random variables which define the column cross section for each simulation.

The value of the steel yield stress, f_{yi}, corresponding to the simulation of order i, is given by

$$f_{yi} = f_{ym}\left(1 + V_{fy}\, Z_i\right) \tag{4}$$

where f_{ym} and V_{fy} are the mean value and the variation coefficient of f_y, and Z_i is a Gaussian random variable with mean equal to zero and variance equal to one.

Z_i numbers are obtained from the following expression

$$Z_i = \left(-2\ln S_i\right)^{1/2} \cos(2\pi S_{i+1}) \tag{5}$$

where S_i and S_{i+1} are two random numbers statistically independent and with uniform distribution in the interval $[0,1]$.

When employing equations (1) to (5), it is admitted that the random variables are independent. Thus, different values of S_i are used to generate these variables.

Spatial variations of the concrete properties can be represented as discontinuous or continuous random fields [1,3,4,13]. The discontinuous representation known as midpoint method is adopted in this work. In this method, the random field over an element is represented by its value at the centroid of the element.

In finite element reliability analyses it may be advantageous to work with two meshes, namely, the finite element mesh and the random field mesh. The former is selected considering the stress gradient while the latter is refined to obtain a satisfactory representation of the random field [14-16]. Generally, the random field mesh is equal to or coarser than the finite element mesh, such that each random field element contains one or more finite elements. This is done to reduce the number of random variables and, consequently, the CPU time. When Monte Carlo method is employed this procedure is unnecessary because the gain in CPU time is small. Then, in this work the random field elements coincide with the finite elements.

A concrete property α, as the modulus of elasticity, the compressive strength or the tensile strength, is represented by a homogeneous Gaussian random field given by

$$\alpha = \alpha_m \left(1 + a(\mathbf{x})\right) \tag{6}$$

where α_m is the mean value of the property and $a(\mathbf{x})$ represents the fluctuations around the mean value, with $\mathbf{x}$ denoting the position vector on the structural domain.

The fluctuating part $a(\mathbf{x})$ has a mean value equal to zero and an autocorrelation function represented by

$$R(d) = E\left[a(\mathbf{x})a(\mathbf{x} + \mathbf{d})\right] \tag{7}$$

where d is the distance between two points $\mathbf{x}$ and $\mathbf{x} + \mathbf{d}$ on the structural domain and $E[\ ]$ symbolizes the expected value.

The autocorrelation function adopted in this work is given by

$$R(d) = V_\alpha^2 \; e^{-\left(d/k\right)^2} \tag{8}$$

where k is the correlation length and V_α is the property variation coefficient.

For $k = 0$ properties are not correlated and when k tends to infinite the correlation is perfect. In these extreme cases the random field is reduced to random variables.

To obtain an one-dimensional random field, the correlation length must be large when compared to the height of the column cross section. The correlation length is taken as $k = 6h_m$, with h_m denoting the mean height of the column cross section. For this value of k, it is possible to disregard the property variations in the same cross section. In fact, only the variations along the column length are significant in this case.

Using the finite element method, concrete properties are taken at the center of each element and are considered constant over the element domain (midpoint method). This procedure requires small elements with respect to the correlation length, as will be shown. Thus, if the column is discretized in NE finite elements, it is necessary to determine NE values of each property α associated to these elements. Consequently, it is necessary to generate NE values of the random variable $a(\mathbf{x}_i)$, where $\mathbf{x}_i$ represents the coordinates at the center of the generic element i.

The correlation characteristics can be specified in terms of the covariance matrix with a generic component given by

$$Cov\left[a_i, a_j\right] = R\left(d_{ij}\right) \tag{9}$$

where d_{ij} is the distance from the center of gravity of element i to the center of gravity of element j.

A vector $\mathbf{a} = \{a_1, a_2, ..., a_{NE}\}^T$ of autocorrelated random variables with mean values equal to zero can be generated as $\mathbf{a} = \mathbf{LZ}$, where $\mathbf{L}$ is the lower triangular matrix obtained by Cholesky's decomposition of the covariance matrix and $\mathbf{Z}$ is a vector containing NE uncorrelated Gaussian random variables obtained by expression (5). If the finite element mesh is excessively fine, the random variables are highly correlated and Cholesky's decomposition of the covariance matrix may become numerically unstable. In this case the random field may be expanded in a Fourier-type series as presented in Reference [13].

After the vector $\mathbf{a}$ has been generated, equation (6) is employed to obtain the concrete properties in each finite element.

To avoid negative values for the Gaussian random variables (steel and concrete properties), values obtained in each simulation are limited to the interval

$$0.05x_m \le x_i \le 1.95x_m \tag{10}$$

where x_i is the generated value and x_m is the mean value of the generic variable x.

In case of resulting some value out off the interval above, the simulation is disregarded. Expression (10) introduces a negligible cutoff in the Gaussian distribution.

Once the stochastic field is defined, the structure is analyzed and different responses are obtained for each simulation. Therefore, expected values and variances of the structural response can be estimated as a function of the sample size (number of simulations). Convergence of the mean value and of the standard deviation of the required structural response will be achieved increasing the number of simulations.

3 VERIFICATION OF THE STRUCTURAL SAFETY

Structural safety may be evaluated comparing its resistance to external loads. The difference between these two values is a measure of the distance to the ultimate limit state. Considering resistance and loads as random variables, it is necessary to formulate the problem in terms of the failure probability.

If F_u is a random variable representing the rupture load of the structure (i.e., its load capacity), then

$$F_u = F_u(X_i), \quad i = 1,\ldots,r \tag{11}$$

where X_i (with $i = 1,\ldots,r$) denotes the basic random variables which contribute to the structural resistance.

If F_s represents the applied loads, the failure probability, p_F, is given by

$$p_F = P(F_s > F_u) \tag{12}$$

and indicates the probability of external actions exceeding the structural resistance.

This problem can be formulated in terms of the safety margin or of the safety coefficient [8]. The first alternative is adopted in this work.

The safety margin, M, is a random variable defined as $M = F_u - F_s$. In this case, failure corresponds to the occurrence of the event $M < 0$.

If $f_M(m)$ represents the probability density function of M, the failure probability is given by

$$p_F = \int_{-\infty}^{0} f_M(m)\,dm \tag{13}$$

When $f_M(m)$ is known, it is simple to calculate the failure probability. The failure probability can be estimated either by means of the Monte Carlo Method or through a reliability index.

In the first procedure, the mean value of the safety margin is $\mu_M = \overline{F}_u - \overline{F}_s$, where $\overline{F}_u$ is the mean failure load obtained through simulations and $\overline{F}_s$ is the mean value of the applied load. Considering that F_u and F_s are independent random variables, the variance of the safety margin is $\sigma_M^2 = \sigma_{FU}^2 + \sigma_{FS}^2$, where σ_{FU}^2 and σ_{FS}^2 are the variances of the failure load and of the applied load, respectively.

Defining the reduced variable $s = (m - \mu_M)/\sigma_M$, the equation (13) can be written as

$$p_F = \int_{-\infty}^{-\beta} f_M(s)\,\sigma_M\,ds \tag{14}$$

where $\beta = \mu_M / \sigma_M$ is the reliability index.

As can be observed, the failure probability reduces as β increases, regardless the distribution $f_M(s)$. Thus, β index is an important measure of the safety level once it is related to the failure probability or, alternately, to the structural reliability.

In an axes system represented by basic random variables, β index indicates the shortest distance from the origin to the failure surface $M = 0$. Therefore, to obtain the reliability index it is necessary to solve a constrained optimization problem. First-order or second-order reliability methods can be used with this purpose and several solution algorithms are available [17,18].

In the second procedure, which is adopted in this work, the reliability index, defined as $\beta = \mu_M / \sigma_M$, can be obtained through simulations. Employing the Monte Carlo method, the structure is analyzed in order to determine its load capacity F_u in each simulation. Mean value and standard deviation of the load capacity can be estimated for a large number of simulations. Mean values and standard deviations of the applied load are given and the reliability index can be calculated as previously described.

4 A MODEL FOR REINFORCED CONCRETE COLUMNS ANALYSIS

In this work, a finite element for plane frames, with two nodes and three degrees of freedom for node, is used to analyze reinforced concrete columns. The nonlinear equations system, due to material and geometrical nonlinearities, is solved iteratively using the BFGS method. Small load increments are applied on the column until that the failure occurs.

Concrete and steel are modeled as nonlinear elastic materials. It is assumed the usual hypothesis of plane sections and a perfect bond between concrete and steel. The analysis is limited to static and short-time loads.

It is assumed that the steel presents a perfect elastic-plastic behavior in tension and in compression with a maximum tensile strain equal to 0.01. The stress-strain diagram for concrete in tension takes into account the tension-stiffening effect by means of a descending branch. The stress-strain relationship for concrete in compression presented in the CEB Codes[5,6] is employed considering the maximum compressive strain as -0.0035.

Three Gauss integration points are considered along the finite element length. At each integration point the concrete cross section is discretized in 30 layers in the height direction. The resistant sectional forces are obtained considering the stresses in the reinforcement and in the center of each concrete layer.

Failure of the cross section localized in a Gauss point is detected by means of the limit strain diagram suggested in the CEB Code/78[5]. This diagram takes into account concrete crushing and excessive elongation of the steel.

The mean values of the modulus of elasticity, E_{cm}, and of the concrete tensile strength, f_{tm}, are given by [6]

$$E_{cm} = 21500\left(\frac{f_{cm}}{10}\right)^{1/3} \text{, MPa} \tag{15}$$

$$f_{tm} = 1.40\left(\frac{f_{cm}}{10}\right)^{2/3} \text{, MPa} \tag{16}$$

where f_{cm} is the mean compressive strength of the concrete.

The steel elasticity modulus is $E_s = 200$ GPa.

5 COMPARISON WITH EXPERIMENTAL RESULTS

Eight slender columns tested in reference [19] are analyzed in this chapter. The cross section, loading and finite element discretization of the columns are indicated in Fig. 1. The finite elements have the same length.

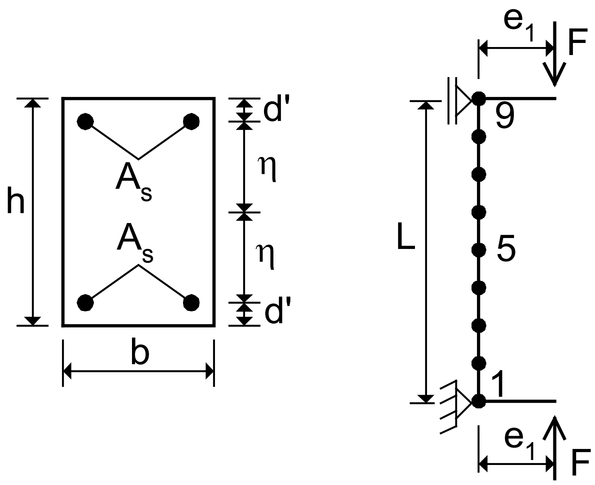

Fig. 1 - Columns geometry and loading

The columns length is $L = 183$ cm and mean dimensions of the cross sections are $b_m = h_m = 7.6$ cm. The steel area in each face of the section is $A_s = 0.71$ cm^2 and $d' = 1.27$ cm ($\eta_m = 2.53$ cm). Four values for the first order eccentricity, e_1, were used in the tests.

Concrete presented a mean compressive strength $f_{cm} = 22$ MPa with a variation coefficient $V_{fc} = 0.10$. The steel reinforcement has a mean yield stress $f_{ym} = 360$ MPa. In the numerical analysis it was assumed that the variation coefficient for this property is $V_{fy} = 0.05$. Values $V_{Ec} = 0.10$ and $V_{ft} = 0.20$ were adopted for the variation coefficients of the modulus of elasticity and of the concrete tensile strength.

Expected values and standard deviations of the rupture load as a function of the number of simulations are presented in Fig. 2 and Fig. 3, respectively.

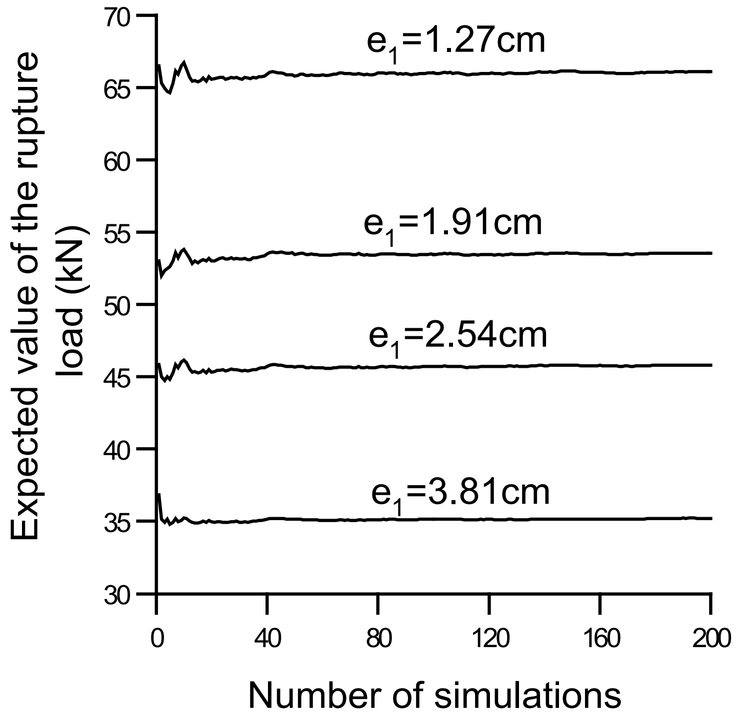

Fig. 2 - Expected values for failure load as a function of the number of simulations

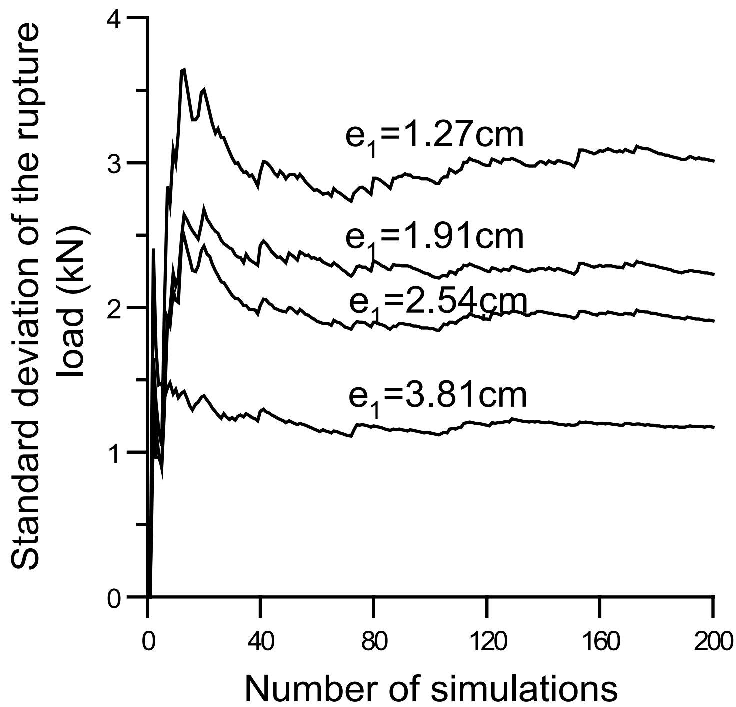

Fig. 3 - Standard deviations for failure load as a function of the number of simulations

Analyzing these figures, it is observed that expected values converge quickly as the size of the sample increases. Standard deviations converge slowly, but when the sample is constituted by results obtained in 200 simulations, its fluctuations can be disregarded.

The influence of the finite element mesh on the variation coefficient of the failure load is shown in Fig. 4. It is observed that results are strongly dependent on the adopted mesh. Convergence it is obtained with eight finite elements when the element length is equal to one-half of the correlation length. Similar results were obtained in others works [3,15].

In a deterministic analysis only three finite elements are necessary to obtain the failure load with accuracy. Thus, the selection of the mesh is a very important task in the finite element reliability analyses.

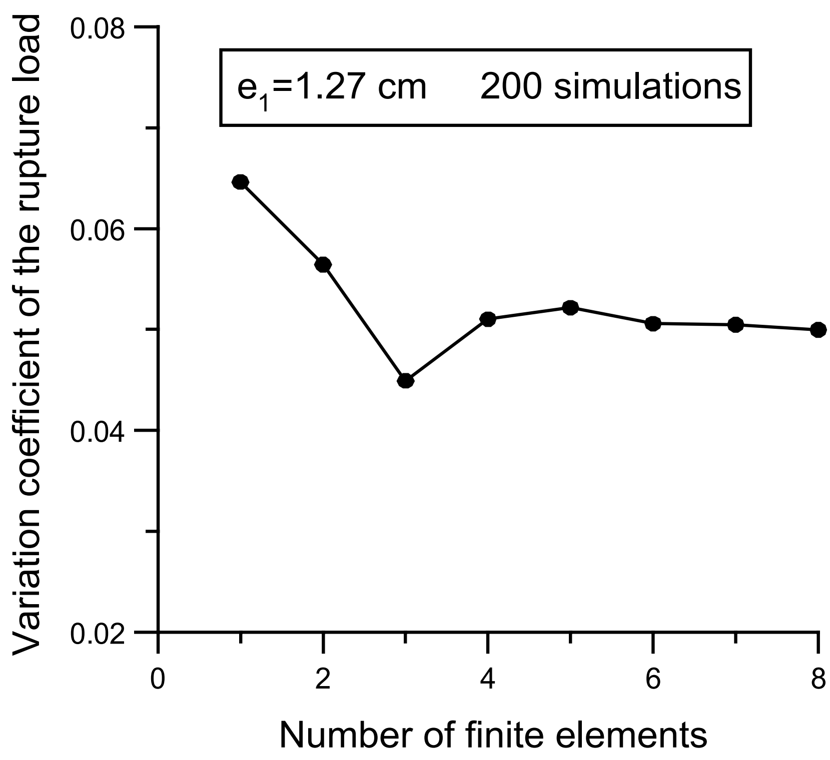

Fig. 4 - Influence of the mesh on the variation coefficient of the failure load

Based on these results, for the following examples the number of finite elements is adopted from the relation $2L/k \geq 8$, where L is the column length and k is the correlation length.

Relations between rupture load and first order eccentricity are presented in Fig. 5. The band corresponding to an occurrence probability of 90%, and the experimental points, are also shown. These results validate the computational model employed.

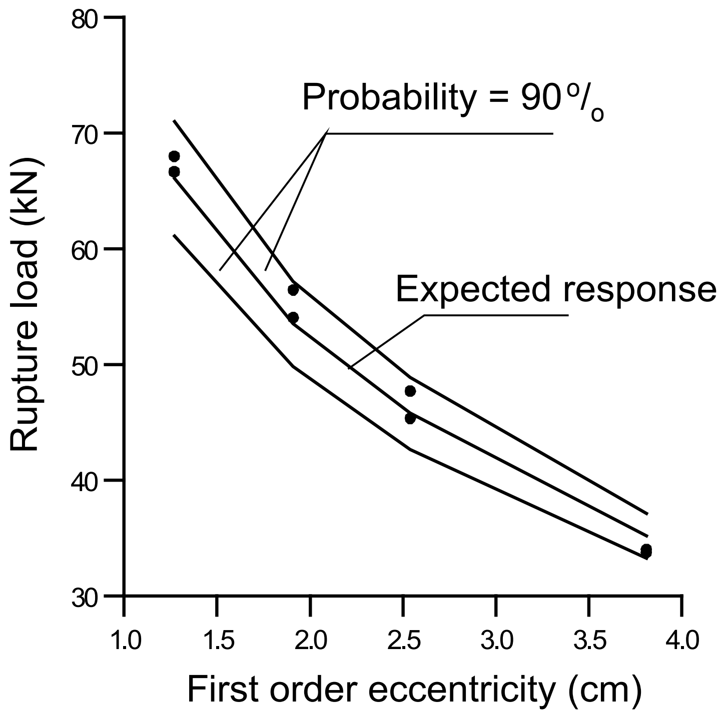

Fig. 5 - Rupture load as a function of the first order eccentricity

6 DETERMINATION OF THE RELIABILITY INDEX

The aim of this chapter is the determination of the reliability index obtained with the design procedures of reinforced concrete columns presented in design codes[5-7]. In these procedures a proportional loading is usually assumed, i.e., the axial load F is applied with an constant eccentricity e_1. Thus, the bending moment, $M_1 = Fe_1$, increases proportionally to the load F. In this case, axial load and bending moment are perfectly correlated.

In reference [12], reliability was evaluated considering three load paths: proportional loading (denoted as path 1), sequential loading with axial force followed by bending moment (denoted as path 2), and sequential loading with bending moment followed by axial force (denoted as path 3). Load paths 1 and 2 failure surfaces are identical, but load path 3 presents a smallest safety domain. Thus, it is concluded that reliability of reinforced concrete columns may be load-path-dependent [12].

This load-path-dependency is not considered in this work in order to be consistent with the design procedures previously mentioned. Thus, it is assumed that the bending moment is proportional to the axial load.

The pin-ended column indicated in Fig.1 is analyzed, considering constant values for the following variables:

$b_m = h_m = 20$ cm (mean dimensions of cross sections);

$\eta_m = 7$ cm (mean position of the steel reinforcement);

$f_{ck} = 20$ MPa (characteristic compressive strength of the concrete);

$f_{yk} = 500$ MPa (characteristic yield stress of the steel);

In design codes, the characteristic strength is defined as the strength with an occurrence probability equal to 5%. Assuming the normal distribution of probability, the mean values of material strengths are given by

$$f_{cm} = \frac{f_{ck}}{1 - 1.645 V_{fc}} \quad ; \quad f_{ym} = \frac{f_{yk}}{1 - 1.645 V_{fy}} \tag{17}$$

where V_{fc} and V_{fy} are the variation coefficients of these properties.

By analogy, assuming the equivalent characteristic axial load, F_k, as one with an occurrence probability equal to 95%, the mean axial load, F_m, is given by

$$F_m = \frac{F_k}{1 + 1.645 V_F} \tag{18}$$

where V_F is the variation coefficient of the load.

The coefficient V_F takes into account uncertainties with respect to actions and uncertainties relative to the employed models (including the representation of loads by an equivalent load as given in equation (18)).

Design values of strengths are $\sigma_{cd} = 0.85 f_{cd}$, for concrete, and f_{yd}, for steel, where

$$f_{cd} = \frac{f_{ck}}{\gamma_c} \ ; \quad f_{yd} = \frac{f_{yk}}{\gamma_s} \tag{19}$$

Design value of the applied load, F_d, is given by

$$F_d = \gamma_f F_k \tag{20}$$

In this work, the partial safety factors are taken as $\gamma_c = 1.4$, $\gamma_s = 1.15$ e $\gamma_f = 1.4$. Variation coefficients for random variables are $V_F = 0.1$; $V_{fy} = 0.05$; $V_{Ec} = V_{fc}$; $V_{ft} = 1.5 V_{fc}$.

The required steel area, A_s, is obtained for the critical cross section of the column considering design values of strengths and the nominal dimensions b_m, h_m and η_m. This steel area is calculated assuming the simplified uniform stress diagram for concrete as suggested by CEB Code [5]. Steel area is calculated in order to equilibrate the axial force F_d and the bending moment $M_d = F_d(e_1 + e_2)$. The expression for the second order eccentricity, e_2, is presented in reference [20] and is given by

$$e_2 = \frac{L^2}{10} \frac{\left(0.0035 + f_{yd}/E_s\right)}{h_m}, \quad \text{for} \quad \nu \leq 0.425 \tag{21}$$

$$e_2 = \frac{L^2}{10} \frac{\left(0.0035 + f_{yd}/E_s\right)}{(\nu/0.425)h_m}, \quad \text{for} \quad \nu > 0.425 \tag{22}$$

where L is the effective length of the column and $\nu = F_d / (b_m h_m f_{cd})$.

After the column is designed, the finite element model is employed to determine its rupture load in each simulation. The reliability index is obtained as previously presented, considering 200 simulations.

In Fig. 6 a histogram of rupture locations along the column axis is shown. The column length is $L = 346$cm, which corresponds to a slenderness ratio $\lambda = 60$. The variation coefficient of the concrete compressive strength is $V_{fc} = 0.2$ and the load parameter is $\nu = 1.0$.

As can be observed, the majority of failed sections are located at half-length of the column as a consequence of the symmetrical loading. However, rupture can occur in any cross section along the column length. This fact demonstrates that for a realistic probabilistic analysis it is necessary to consider the spatial variations of the concrete properties. Simplified procedures that predetermine the cross section where failure occurs are not satisfactory for reliability analysis because this predetermination may be a strong deterministic component.

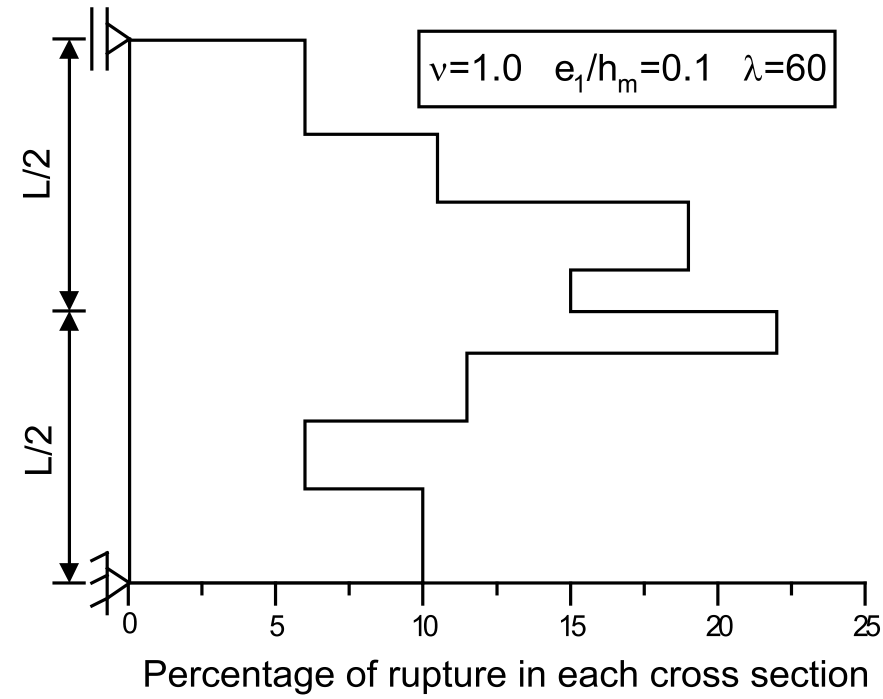

Fig. 6 - Histogram of the rupture location

Variations of the reliability index with the relative first order eccentricity e_1/h_m , are shown in Fig. 7. Different axial loads are defined by the parameter ν. Limits for e_1/h_m and ν correspond to a maximum percentage of steel reinforcement established in design codes [6,7].

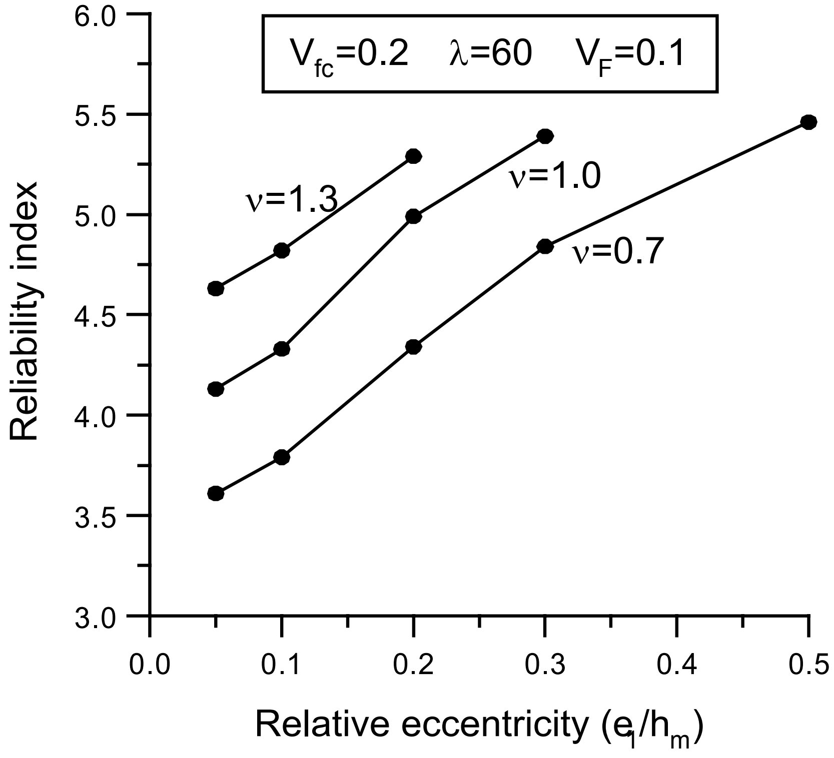

Fig. 7 - Influence of the relative first order eccentricity in the reliability index

Observing Fig. 7, it may be verified that the reliability index is smaller for small values of the relative first order eccentricity. Furthermore, the reliability index is greater for large axial loads (defined by the load parameter ν). This happens due to the increase of the steel area when axial load and/or first order eccentricity rise. It can be concluded that the partial safety factors method introduces different safety levels depending on the load conditions.

In Fig. 8 the influence of the variation coefficient of the concrete compressive strength in the reliability index is shown. The reliability index decreases as the variability of the concrete compressive strength rises, as it would be expected. Thus, safety is very dependent on the quality control employed in the concrete production and structure execution.

To take into account the variability of concrete properties, the factor γ_c can be selected according to the quality control. For example, the standard value $\gamma_c = 1.4$ can be adopted for a construction of average quality in which the variation coefficient V_{fc} is contained between 0.15 and 0.20. For other quality controls, γ_c can be adjusted to obtain the same safety level. Similarly, the factor γ_f can be modified to consider different loading combinations.

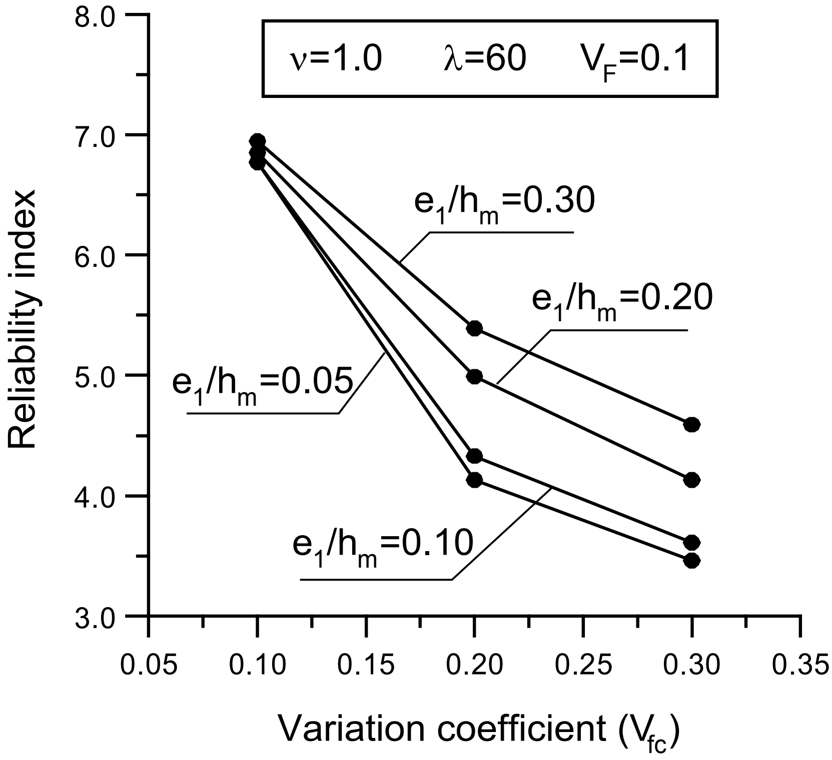

Fig. 8 - Influence of the variability of the concrete compressive strength in the reliability index

In Fig. 9, the expected values of the failure load as a function of the slenderness ratio λ are shown. In the same Figure the results obtained from the deterministic analysis (with the mean values of the random variables) are presented.

It may be verified that the expected values of the rupture load are smaller than those obtained from the deterministic analysis. From this figure it may be concluded that the rupture load decreases as the slenderness of the column grows. Then, the deterministic analysis leads to the interpretation that the safety is greater for shorter columns.

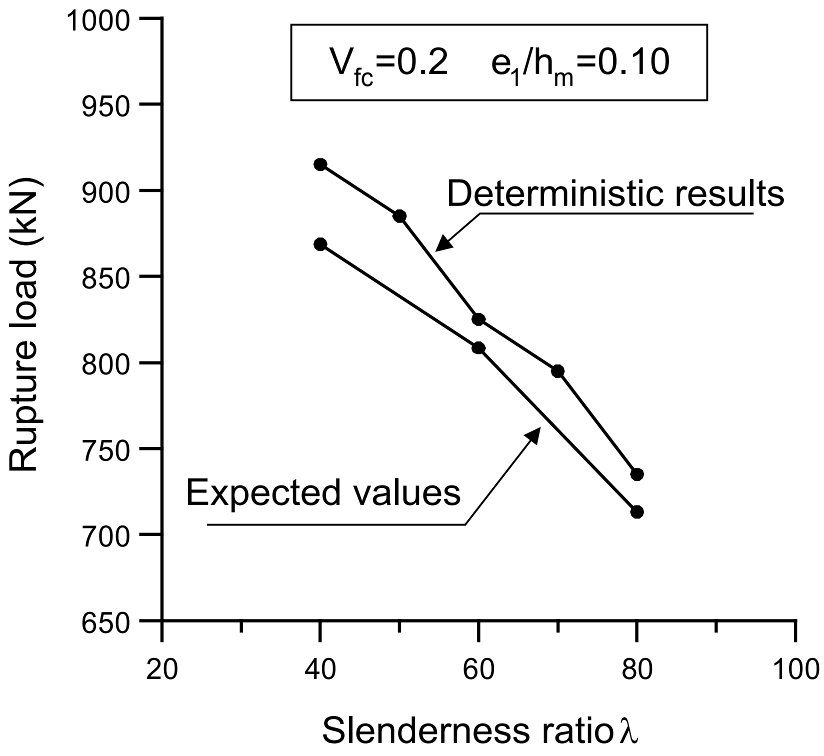

Fig. 9 - Influence of the slenderness in the rupture load

In Fig. 10 the influence of the slenderness in the reliability index is presented. This Figure shows that slenderer columns are those presenting greater safety, which is contradictory with results obtained from the deterministic analysis. This occurs because the steel area obtained in the design increases as slenderness rises. Thus, it can be concluded that the steel ratio has a favorable effect on reliability (despite the increase of the slenderness).

This is justified by the fact that both mean value and standard deviation of the rupture load decrease as slenderness rises. For the range of λ and ν shown in Fig. 10, it was verified that the variation coefficient of the failure load decreases as slenderness increases. Thus, the reliability index increases with the growth of the slenderness.

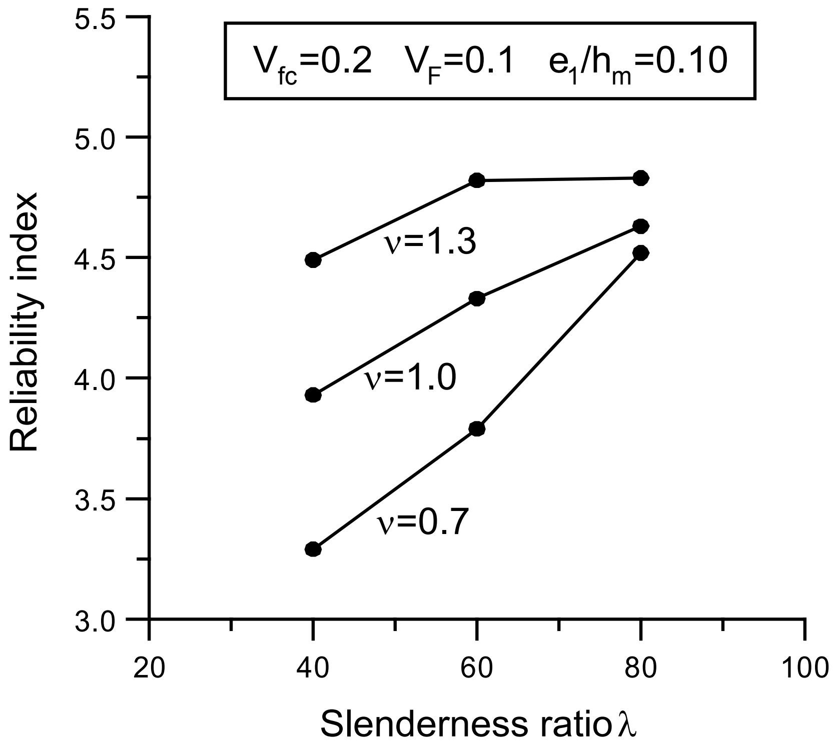

Fig. 10 - Influence of the slenderness in the reliability index

Fig. 11 presents the variation of the reliability index with the correlation length of the random field for concrete properties. It is observed that the reliability index is strongly affected by the correlation length. The reliability index decreases with the rise of the correlation length until reaching a minimum value. However, this minimum value is not coincident with the reliability index obtained when spatial variations of the concrete properties are disregarded. The minimum value of the reliability index is associated to a finite correlation length. Therefore, the assumption that concrete properties are random variables (instead of random fields) does not lead to a lower reliability index in this case.

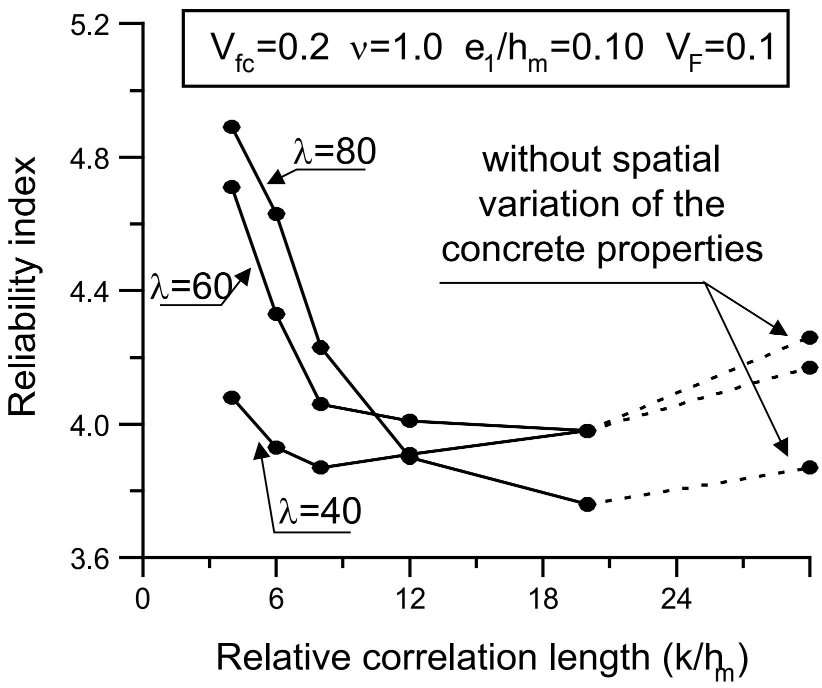

Fig. 11 - Influence of the correlation length on reliability

As it was shown in these examples, reliability of reinforced concrete columns depends on several parameters such as: design value of the applied load, first order eccentricity, slenderness ratio, variability of steel and concrete properties (including the correlation length of the random field), variability of loads and geometrical properties.

Design codes adopt constant partial safety factors that, for the most unfavorable conditions, introduce a satisfactory reliability. However, these safety factors can be modified in order to obtain a uniform reliability. For example, factors γ_c and γ_s can be adjusted to consider the variability of concrete and steel properties related to the quality control. Factor γ_f must include load variability and the model uncertainties. Other parameters can be considered by adjusting the model.

7 CONCLUSIONS

In this work, the probabilistic finite element method was used to analyze reinforced concrete columns. Dimensions of the column cross section, yield stress and position of the steel reinforcement, and axial load, were considered as random variables. Concrete properties were represented as homogeneous Gaussian random fields. Results obtained are limited to the proportional loading.

The Monte Carlo method was employed to obtain expected values and standard deviations of the column rupture load. Results have indicated a good agreement with available experimental data. Afterwards, the computational model was used to evaluate the safety level obtained with the usual design procedures of reinforced concrete columns.

It was shown that in order to carry out a realistic safety analysis it is necessary to consider spatial variations of the concrete properties (random fields). When concrete properties are considered as random fields, it is verified that failure may occur in any cross section along the column length. Procedures, which consider concrete properties as single random variables, are unsuitable for safety analysis because they pre-establish the cross section where failure will occur and this may be a strong deterministic component.

On the other hand, random fields introduce an important mesh-dependency in a probabilistic finite element analysis. This mesh-dependency is dictated by the correlation length of the random field. In this work it was verified that the element length must be equal or smaller that one-half of the correlation length.

Furthermore, the correlation length has a significant effect on reliability. The minimum value of the reliability index is associated to a finite correlation length. Then, assuming that concrete properties are random variables (case in which the correlation length tends to infinite) is not a safe procedure to establish partial safety factors for a desired reliability.

This study has shown that reliability of reinforced concrete columns depends on several parameters related to the design method as well as to the variability of basic variables. The main parameters of the design method are the first order eccentricity, slenderness ratio and the design value of the applied load. Increasing any of these parameters implicates in an increase of the steel reinforcement ratio and this has a favorable effect on reliability.

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