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Comparative study of the simplified methods of Eurocode 2 for second order analysis of slender reinforced concrete columns

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ABSTRACT

Usually, reinforced concrete design codes indicate only one simplified method for second order analysis of slender columns. The Eurocode 2 (EC2), on the other hand, adopts two simplified methods: one based on nominal stiffness and other based on nominal curvature. It would be desirable that both methods could provide similar solutions. However, this is not the case, as shown in this paper. On the contrary, the two EC2 simplified methods can provide very different results, leaving the engineer uncertain about which method he should use. The objective of this work is to compare these two simplified methods presenting the contradictions between them. Several experimental results available in the literature have been analysed and compared. The method based on the nominal curvature showed to be the most accurate; therefore, it is suggested to be used.

1. Introduction

The design of reinforced concrete slender columns requires the consideration of the material and geometric non-linearities. Material non-linearity is due to the non-linear behaviour of concrete, including cracking, as well as the yielding of the reinforcement. Geometric non-linearity arises from the need to verify the equilibrium in the deformed structure. Bending moments in the initial undeformed configuration of the column axis are called first order moments. The additional moments caused by deformations are called second order moments.

Due to the importance of the columns for structural stability, design codes [1–4] require that such additional second order effects be considered in the design of columns. Only in very short columns it is allowed to ignore the second order effects.

When the column is slender and the second order effects are very important, it is necessary to perform a complete non-linear analysis, where non-linearities are considered appropriately. This analysis requires the use of numerical methods, such as the finite element method or the finite difference method, associated with iterative and incremental techniques for solving the system of non-linear equations [5–8]. In cases of columns in usual buildings, it is allowed to adopt simplified methods without the need of this complex non-linear analysis.

EC2 [4] adopts two simplified methods for second order analysis of slender reinforced concrete columns: a method based on nominal stiffness and a method based on nominal curvature. The first method is similar to the moment magnification procedure adopts by ACI Building Code [1]. The second method is the only simplified method recommended by CEB FIP Model Code 1990 [2] and by FIB Model Code 2010 [3].

The method based on nominal stiffness may be used for both isolated members and whole structures, if nominal stiffness values are estimated appropriately. The method based on nominal curvature is mainly suitable for isolated members [4].

In both simplified methods, when the column is subjected to different first order end moments M_{01} and M_{02}, an equivalent constant first order moment M_{0e} is adopted. The equivalent first order moment is given by

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02} \tag{1}$$

where M_{01} and M_{02} have the same sign if they give tension on the same side, otherwise opposite signs. Furthermore, $|M_{02}| \geq |M_{01}|$.

The two simplified methods of EC2 have been extensively studied in order to improve their accuracy [9–11]. In these studies alternative formulas for calculating the nominal stiffness or nominal curvature are proposed, which are derived from non-linear analysis.

The purpose of this paper is to analyse these two simplified methods as they are presented in EC2. It is not the intention of this work to propose alternative formulas for design. Making a comparative analysis of the design equations, contradictions between the two methods are shown. Through the analysis of a series of experimental results available in the literature, it is possible to evaluate the accuracy of the two methods. If both methods provide approximately equal solutions, it would be acceptable for them to be suggested in EC2. However, due to the large difference in results obtained with the two simplified methods it is not appropriate to include them in the same design code.

As conclusion of this study, it is suggested to adopt the method

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based on nominal curvature for analysis of columns subjected to short term loads. For columns subjected to sustained loads, it is suggested to consider the additional creep eccentricity adopted in CEB FIP Model Code 1990 [2]. The study is limited to columns subjected to compression and uniaxial bending.

2. Method based on nominal stiffness

In this method, the total design moment M_d, including second order moment, is given by $M_d = \psi_1 M_{0e}$, where ψ_1 is the magnification factor obtained from a linear analysis and M_{0e} is the equivalent first order moment including the effects of imperfections. According EC2, ψ_1 may be expressed as

$$\psi_1 = 1 + \frac{\beta}{(N_{cr}/N_d) - 1} \tag{2}$$

where β is a factor that depends on distribution of the moments, N_d is the design axial load and N_{cr} is the buckling load based on nominal stiffness EI.

For columns with constant cross-section and axial load, $\beta = \pi^2/c_0$, where c_0 depends on the distribution of the first order moment. For a constant first order moment, or when an equivalent moment is adopted, $c_0 = 8$. Eq. (2) can only be used if $N_d < N_{cr}$.

For a column with constant cross-section, the buckling load is given by

$$N_{cr} = \frac{\pi^2 EI}{l_e^2} \tag{3}$$

where l_e is the effective length.

For a pin-ended column, l_e is the actual length of the column. For braced members, the effective length may be reduced to take into account the stiffness of the beams connecting with the column.

The nominal stiffness EI is given by

$$EI = \left(\frac{k_1 k_2}{1 + \varphi_{ef}}\right) E_{cd} I_c + E_s I_s \tag{4}$$

where $E_s = 200$ GPa is the modulus of elasticity of reinforcement and E_{cd} is the design value of the modulus of elasticity of concrete, obtained by

$$E_{cd} = \frac{22}{\gamma_E}\left(\frac{f_{ck} + 8}{10}\right)^{0.3}, \text{ GPa} \tag{5}$$

where $\gamma_E = 1.2$ and f_{ck} is the characteristic compressive strength of concrete in MPa.

The factor k_1 depends on concrete strength and is given by

$$k_1 = \sqrt{f_{ck}/20}\,, \ \text{(with } f_{ck} \text{ in MPa)} \tag{6}$$

The factor k_2 depends on relative axial force ν and slenderness ratio λ, being obtained by

$$k_2 = \frac{\nu\,\lambda}{170} \le 0.20 \tag{7}$$

with $\nu = N_d/(A_c f_{cd})$, where $f_{cd} = f_{ck}/\gamma_c$ is the design compressive strength of concrete and A_c is the area of the column cross-section; and $\lambda = l_e/i$, where i is the radius of gyration of the uncracked concrete cross-section. The partial safety factor for concrete compressive strength is taken as $\gamma_c = 1.5$.

The effective creep ratio φ_{ef} is defined as

$$\varphi_{ef} = \varphi_\infty \frac{M_{0qp}}{M_{0e}} \tag{8}$$

where: $\varphi_\infty =$ final creep coefficient; $M_{0qp} =$ first order bending moment in quasi-permanent load combination (serviceability limit states); $M_{0e} =$ first order bending moment in design load combination (ultimate limit states).

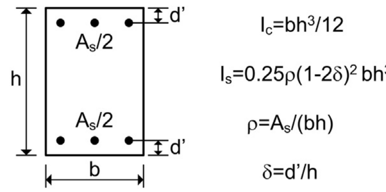

Fig. 1. Rectangular section with symmetrical reinforcement.

In Eq. (4), I_c is the moment of inertia of concrete cross-section and I_s is the second moment of area of reinforcement with respect to the centre of the concrete area. Fig. 1 shows these properties for a rectangular section with two layers of symmetric reinforcement. Bending occurs in the direction of the height h of the cross-section.

The magnification factor ψ_1 given in Eq. (2) tends to infinity when $N_d = N_{cr}$. In practice, one must have $N_d << N_{cr}$ to avoid very high values of ψ_1. For a cross-section with fixed dimensions, a minimum reinforcement ratio $\rho_{\min}$ must be specified, as shown below.

Considering the expressions of I_c and I_s given in Fig. 1, Eq. (4) can be written as

$$EI = \left[\frac{k_1 k_2 E_{cd}}{(1 + \varphi_{ef})} + 3\rho(1 - 2\delta)^2 E_s\right]\frac{bh^3}{12} \tag{9}$$

The slenderness ratio for the rectangular section of Fig. 1 is given by $\lambda = l_e\sqrt{12}/h$. Thus, the buckling load given in Eq. (3) can be written as

$$N_{cr} = \frac{\pi^2}{\lambda^2}\left[\frac{k_1 k_2 E_{cd}}{(1 + \varphi_{ef})} + 3\rho(1 - 2\delta)^2 E_s\right] bh \tag{10}$$

Considering $\psi_1 \le 5$, for example, Eq. (2) gives $N_{cr} \ge (1 + 0.25\beta)N_d$. Using Eq. (10) and substituting $N_d = \nu bh f_{cd}$, the following expression is obtained for the minimum reinforcement ratio:

$$\rho_{\min} = \frac{1}{3(1 - 2\delta)^2 E_s}\left[\frac{(1 + 0.25\beta)\nu\lambda^2 f_{cd}}{\pi^2} - \frac{k_1 k_2 E_{cd}}{(1 + \varphi_{ef})}\right] \ge 0 \tag{11}$$

The minimum reinforcement area is given by $A_{s,\min} = \rho_{\min} A_c$.

In order to use Eq. (4), it is necessary to know the column reinforcement. Thus, the design of the cross-section subjected to the total moment M_d combined with the axial force N_d requires the use of an iterative process. In each iteration, after the calculation of the steel area A_s, the stiffness EI is updated and a new total moment must be calculated. The design of the cross-section subjected to combined flexure and axial force is made according to EC2 recommendations, assuming a rectangular stress distribution for compressed concrete.

Initially, it is determined the steel area A_{s1} for the axial force N_d combined with the first order moment M_{0e}. It must be ensured that $A_{s1} \ge \rho_{\min} A_c$. With this steel area, stiffness EI is obtained and the magnification factor ψ_1 is calculated through Eq. (2). Carrying out a new reinforcement calculation for the axial force N_d and the moment $M_d = \psi_1 M_{0e}$, the steel area A_{s2} is obtained. The requested steel area A_s is in the interval $A_{s1} \le A_s \le A_{s2}$.

For a generic value A_{si} of the steel area, one can calculate the design resistant moment M_{Rd} that acts together with the design axial force N_d in the ultimate limit state. The total bending moment is $M_{Sd} = \psi_1(A_{si}) M_{oe}$, where $\psi_1(A_{si})$ is the magnification factor obtained considering the steel area A_{si}. The solution A_s of this problem is such that $f(A_s) = M_{Rd} - M_{Sd} = 0$ and can be obtained through the bisection iterative method, as illustrated in Fig. 2. The convergence of the process is considered when $|M_{Rd} - M_{Sd}|/M_{oe} < 0.01$.

As a simplified alternative, provide $\rho \ge 0.01$, can be adopted

$$EI = \left(\frac{0.3}{1 + 0.5\varphi_{ef}}\right) E_{cd} I_c \tag{12}$$

This simplified alternative may be suitable as a preliminary step,

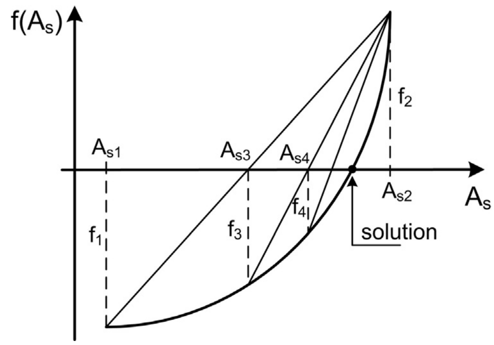

Fig. 2. Bisection iterative method.

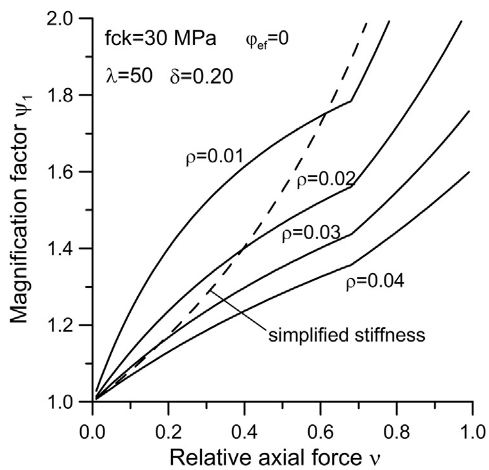

Fig. 3. Magnification factor ψ_1 for different reinforcement ratios.

followed by a more accurate calculation according to Eq. (4).

Fig. 3 shows the magnification factor ψ_1 obtained using Eq. (4) and the simplified stiffness given in Eq. (12). It can be observed that the magnification factor ψ_1 increases when the relative axial force ν increases. On the other hand, it decreases by increasing reinforcement ratio ρ. The magnification factor ψ_1 is independent of the first order moment M_{0e}.

The magnification factor ψ_1 increases very rapidly when the load N_d approaches the buckling load N_{cr}. In fact, this is what is verified through the elastic solution of the column subjected to an eccentric load. However, in several cases the buckling load is underestimated resulting in an excessive and unrealistic value for the magnification factor ψ_1 as will be shown by comparison with results of experimental tests. In several of these tests the column failure load was greater than the buckling load calculated with Eq. (3), which prohibits the use of this simplified method, as shown in Tables 2, 3.

3. Method based on nominal curvature

In this method, the total design moment is $M_d = M_{0e} + M_2$, where M_{0e} is the equivalent first order moment, including the effects of imperfections, and $M_2 = N_d\, e_2$ is the second order moment, with e_2 being the second order eccentricity. Taking the first order eccentricity as $e_1 = M_{0e}/N_d$ then $M_d = N_d\, e_d$, where $e_d = e_1 + e_2$ is the total eccentricity for the column design.

The second order eccentricity is given by

$$e_2 = \frac{l_e^2}{c}\chi \tag{13}$$

where χ is the curvature and c is a factor depending on the curvature distribution.

For constant cross-section, $c = 10\,(\approx\pi^2)$ is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment). The value π^2 corresponds to a sinusoidal curvature distribution. The value for constant curvature is 8. In the examples presented in this paper, it is adopted $c = 10$.

For members with constant symmetrical cross-section (including reinforcement), it is adopted

$$\chi = K_r K_\varphi\left(\frac{\varepsilon_{yd}}{0.45d}\right) \tag{14}$$

where:

$K_r =$ correction factor depending on axial load; $K_\varphi =$ factor for taking account of creep; $\varepsilon_{yd} = f_{yd}/E_s =$ design yield strain of reinforcement; $f_{yd} = f_{yk}/\gamma_s =$ design yield strength of reinforcement; $f_{yk} =$ characteristic yield strength of reinforcement; $\gamma_s = 1.15 =$ partial safety factor for reinforcement; $E_s =$ modulus of elasticity of reinforcement; $d =$ effective depth.

The factor K_r should be taken as

$$K_r = \frac{1 + \omega - \nu}{0.6 + \omega} \le 1 \tag{15}$$

where $\nu = N_d/(A_c f_{cd})$ is the relative axial force and $\omega = A_s f_{yd}/(A_c f_{cd})$ is the mechanical reinforcement ratio.

The factor K_φ should be taken as

$$K_\varphi = 1 + \left(0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150}\right)\varphi_{ef} \ge 1 \tag{16}$$

where f_{ck} is given in MPa, λ is the slenderness ratio and φ_{ef} is the effective creep ratio given in Eq. (8).

It is observed that K_φ decreases with increasing slenderness ratio λ, indicating that creep can be neglected for very slender columns. For $\lambda \ge 52.5 + 0.75 f_{ck}$ it results $K_\varphi = 1$, indicating that creep will not be considered for these columns. This is not correct and contradicts EC2 itself, which allows us to ignore the effects of creep if $\varphi_\infty \le 2$, $\lambda \le 75$ and $e_1/h \ge 1$ simultaneously. Eq. (16) needs to be revised.

The CEB-FIP Model Code 1990 [2] and the FIB Model Code 2010 [3] adopt this same EC2 formulation in order to consider second order effects in columns design. However, there are differences in the consideration of the creep effects. In FIB Model Code 2010 the long term deformation due to creep is taken into account as pre-curvature of the cross-section. In CEB-FIP Model Code 1990 the creep effects are introduced as an additional eccentricity e_c given by

$$e_c = e_1\left[\exp\left(\frac{\varphi_{ef}}{N_{cr}/N_{Sg} - 1}\right) - 1\right] \tag{17}$$

where N_{Sg} denotes the axial load under the quasi-permanent combination of actions.

The critical Euler-load N_{cr} is given in Eq. (3) considering $EI = E_{cm} I_c$, where E_{cm} is the modulus of elasticity of concrete which may be obtained from Eq. (5) by making $\gamma_E = 1$. The second order eccentricity is obtained with the use of Eqs. (13)–(15), considering $K_\varphi = 1$. The total eccentricity is $e_d = e_1 + e_2 + e_c$ and the total moment is $M_d = N_d\, e_d$.

Defining a new magnification factor $\psi_2 = M_d/M_{0e}$ and substituting $M_d = N_d\, e_d$ and $M_{0e} = N_d\, e_1$, results $\psi_2 = e_d/e_1$. It is observed that in this method the magnification factor depends on the first order eccentricity (or the first order moment), which does not occur with the method based on nominal stiffness.

It should be noted that to calculate the second order eccentricity e_2 it is necessary to know the column reinforcement. The dimensioning of the cross-section subjected to the total moment $M_d = N_d\, e_d$ and the axial force N_d requires the use of an iterative process. In each iteration, after the calculation of the steel area A_s, the total eccentricity e_d is updated and a new total moment must be calculated. Since $M_d = \psi_2 M_{0e}$, it is possible to employ the same bisection iterative method described above. The requested steel area A_s is in the interval $A_{s1} \le A_s \le A_{s2}$,

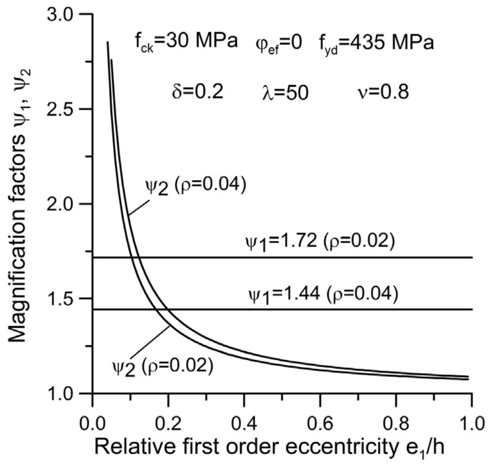

Fig. 4. Magnification factors for the two simplified methods of EC2.

where A_{s1} is the area obtained for $K_r = 0$ and A_{s2} is the area obtained considering $K_r = 1$. To avoid this iterative process, one can take $K_r = 1$ as a conservative simplification.

Fig. 4 compares the magnification factors ψ_1 and ψ_2 obtained using the two simplified methods adopted by EC2. It can be seen that the magnification factor ψ_2 decreases with increase of the relative first order eccentricity e_1/h. The magnification factor ψ_1 is independent of e_1/h. On the other hand, the magnification factor ψ_2 increases with the increase of the reinforcement ratio ρ, unlike what happens with the factor ψ_1. Therefore, there is a clear contradiction between these two simplified methods.

The variation of the magnification factors as a function of the relative axial force ν is shown in Fig. 5. It can be observed that the two magnification factors have completely different behaviours. While the factor ψ_1 increases with increasing axial force, the opposite occurs with the factor ψ_2. This shows another serious divergence between the two simplified methods.

4. Comparison with experimental results

The magnification factors ψ_1 and ψ_2 obtained with the simplified methods are compared with the magnification factor ψ_{exp} determined from tests performed by other authors. In all tests, the columns were subjected to compression and uniaxial bending. In cases where the column was submitted to different first order end moments, the equivalent first order moment M_{0e} was considered.

For each column all variables necessary to compute ψ_1 and ψ_2 are known. The failure load obtained in the experimental test is N_u and the first order moment is $M_{0u} = N_u e_1$, where e_1 is the equivalent first order eccentricity used in the test. The resistant moment M_{Ru} is determined through the resistance analysis of the column cross-section subjected to the axial force N_u. This analysis is made according to EC2 recommendations, assuming a rectangular stress distribution for compressed concrete. Therefore, the experimental magnification factor is given by $\psi_{exp} = M_{Ru}/M_{0u}$. In this sectional resistance analysis, the partial safety factors γ_E, γ_c and γ_s are taken with values equal to 1.0. All tested columns have a rectangular section as shown in Fig. 1.

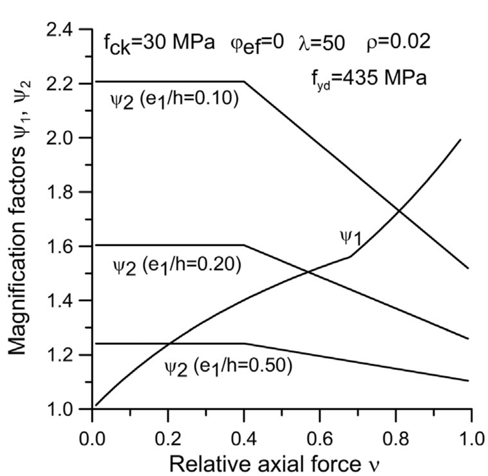

Fig. 5. Magnification factors as a function of the relative axial force.

The accuracy of the simplified methods is verified through the relation $R = \psi_{teo}/\psi_{exp}$, where ψ_{teo} is the theoretical magnification factor. The following convention is used to distinguish the simplified methods analysed:

- Method 1 = method based on nominal stiffness with EI obtained from Eq. (4).
- Method 2 = method based on nominal stiffness with EI obtained from Eq. (12).
- Method 3 = method based on nominal curvature with K_r obtained from Eq. (15) and K_φ from Eq. (16).
- Method 4 = method based on nominal curvature with $K_r = 1.0$ and K_φ obtained from Eq. (16).
- Method 5 = method based on nominal curvature with K_r obtained from Eq. (15), $K_\varphi = 1.0$ and creep effects given by Eq. (17).
- Method 6 = method based on nominal curvature with $K_r = 1.0$, $K_\varphi = 1.0$ and creep effects given by Eq. (17).

Table 1 shows summary information about the columns. Full details may be obtained in the references listed in the table. The tests comprise a total of 115 pin-ended columns, being 83 columns with equal moments at both ends ($M_{01}/M_{02} = 1.0$) and 32 columns with unequal moments ($M_{01}/M_{02} \neq 1.0$). Of these 83 columns with equal first order end moments, 20 are subjected to sustained load. All the other 95 columns were submitted to short term load. For the tests reported in reference [12], the concrete strength is based on prisms with the same cross-section of the columns. For the other tests, the cylinder strengths are considered.

5. Results for columns under sustained load

Goyal and Jackson [12] tested 20 columns under sustained load, being 3 columns with slenderness ratio $\lambda = 55$, 14 columns with $\lambda = 83$ and 3 columns with $\lambda = 125$. The range of the main parameters is given in Table 1.

Table 2 presents the results obtained with the six simplified methods. This table shows the mean value R_m, the standard deviation σ_R and the coefficient of variation $V_R = \sigma_R/R_m$ of the ratio $R = \psi_{teo}/\psi_{exp}$. The value n is the number of columns that are possible to analyse with a simplified method. It is observed that not all 20 columns could be analysed with the methods based on nominal stiffness because the

Table 1
Columns used for checking the simplified methods (parameter range: min-max).

Parameters	Goyal and Jackson [12]	Goyal and Jackson [12]	Melo [13]	Kim and Yang [14]	Leite et al. [15]
columns	20	26	21	16	32
e_1/h	0.17–0.50	0.17–0.50	0.05–0.50	0.30	0.04–0.32
ν	0.15–0.65	0.16–0.67	0.12–0.57	0.10–0.45	0.09–0.77
λ	55–125	55–125	58–87	10–104	69–104
f_{ck}	19.9–23.6	19.9–23.6	33.9–45.8	25.5–86.2	29.5–93.2
φ_{ef}	0.8–1.6	0	0	0	0
ρ%	1.7–2.4	1.7–2.4	1.57	2.0–4.0	2.26–3.39
M_{01}/M_{02}	1.0	1.0	1.0	1.0	−0.5; 0.0; 0.5
loading	sustained	short term	short term	short term	short term

Table 2
Results for columns under sustained load.

Method	n	R_m	σ_R	V_R
1	19	1.79	0.71	0.40
2	13	3.17	1.88	0.59
3	20	0.91	0.09	0.10
4	20	0.93	0.12	0.13
5	20	1.02	0.09	0.09
6	20	1.03	0.12	0.12

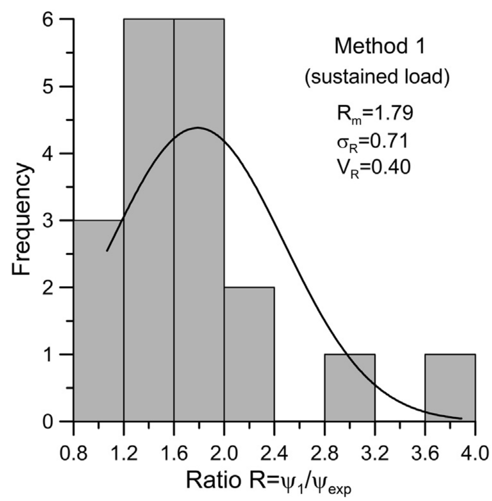

Fig. 6. Histogram of $R = \psi_1/\psi_{\text{exp}}$ for method 1 (sustained load).

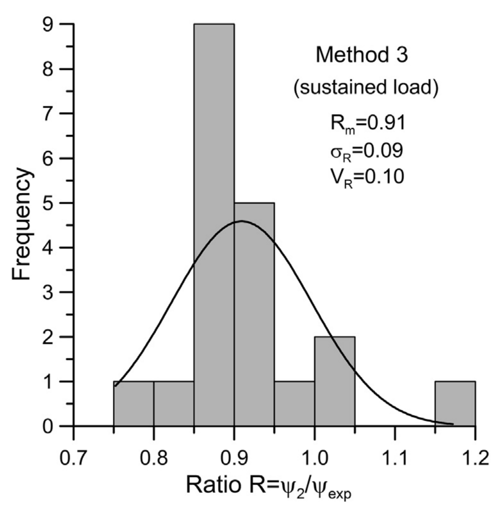

Fig. 7. Histogram of $R = \psi_2/\psi_{\text{exp}}$ for method 3 (sustained load).

failure load N_u observed in the test was greater than the buckling load N_{cr}.

Table 2 clearly shows that the method based on nominal stiffness is excessively conservative. The mean values $R_m = 1.79$ and $R_m = 3.17$ make this method unacceptable. Moreover, this method is applicable only when the design load is small compared to the buckling load. Very slender or heavily loaded columns should not be analysed by this method.

The method based on nominal curvature, as presented in EC2, provides good results but is unconservative. This is because Eq. (16) provides $K_\varphi = 1.0$ for columns with $\lambda = 83$ and $\lambda = 125$. Thus, the creep effects are not included in this method.

On the other hand, a conservative design is obtained if the effects of creep are included through the additional eccentricity given in Eq. (17).

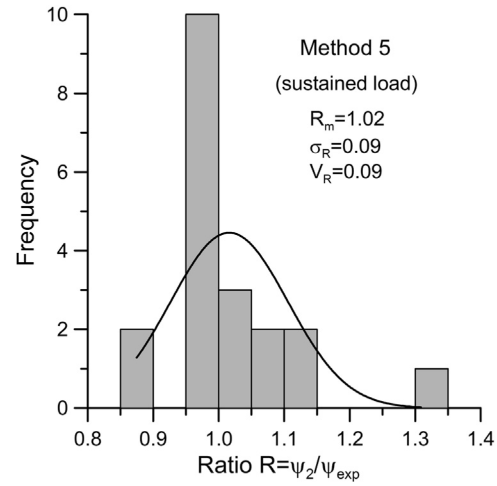

Fig. 8. Histogram of $R = \psi_2/\psi_{\text{exp}}$ for method 5 (sustained load).

Table 3
Results for columns under short term load.

Method	n	R_m	σ_R	V_R
1	69	2.05	1.35	0.66
2	61	1.56	0.87	0.56
3	95	1.31	0.41	0.31
4	95	1.37	0.50	0.36

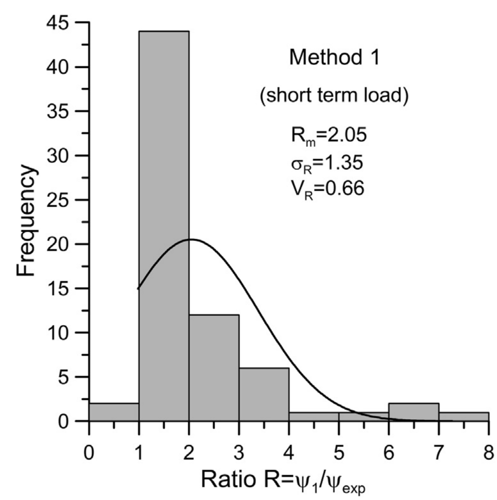

Fig. 9. Histogram of $R = \psi_1/\psi_{\text{exp}}$ for method 1 (short term load).

The mean value R_m is very close to 1.0 in the two options for calculating the factor K_r (Method 5 and Method 6). In addition, the coefficient of variation V_R is very small with any of these options for calculating K_r.

Comparing Method 1 with Method 2, there is a large difference in the mean value R_m. This is because the magnification factor ψ_1 depends on the stiffness EI which is strongly influenced by reinforcement ratio. On the other hand, the mean values R_m obtained with the methods based on nominal curvature are little dependent on the definition of the factor K_r. In this way, it can be adopted $K_r = 1.0$ avoiding the use of the iterative process described above.

Figs. 6–8 show the histograms of the ratio $R = \psi_{teo}/\psi_{\text{exp}}$ obtained with the methods 1, 3 and 5.

6. Results for columns under short term load

The six simplified methods are compared with tests of 95 columns

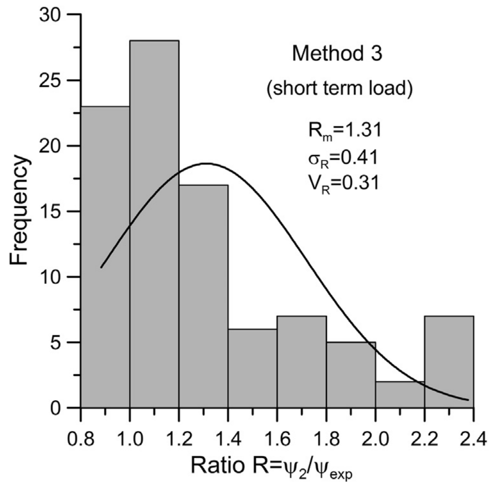

Fig. 10. Histogram of $R = \psi_2/\psi_{\mathrm{exp}}$ for method 3 (short term load).

subjected to short term loads. The range of the main parameters is given in Table 1. Since creep is not included in the analysis, methods 5 and 6 are the same as methods 3 and 4, respectively.

Table 3 shows the results obtained with the simplified methods. It should be noted that not all 95 columns could be analysed with the methods based on nominal stiffness because the failure load N_u observed in the test was greater than the buckling load N_{cr} or the magnification factor ψ_1 was excessive ($\psi_1 > 25$).

Table 3 shows that the method based on nominal stiffness is very conservative. The mean value $R_m = 2.05$ indicates that this method is uneconomical. Moreover, this method is applicable only when the design load is small in relation to the buckling load or the column is not very slender. Again, there is a large difference in the mean values R_m obtained with methods 1 and 2. This indicates a discrepancy between the expressions for the stiffness EI provided by the EC2, as was already shown in Fig. 3. The mean values R_m obtained with the methods based on nominal curvature are little dependent on the definition of the factor K_r and can be adopted $K_r = 1.0$.

Figs. 9 and 10 show the histograms of the ratio $R = \psi_{teo}/\psi_{\mathrm{exp}}$ obtained with the methods 1 and 3.

7. Conclusions

The main reinforced concrete design codes indicate only one simplified method for second order analysis of slender columns. In contrast, the Eurocode 2 adopts two simplified methods: a method based on nominal stiffness and a method based on nominal curvature. By finding two distinct methods suggested in the same design code, the engineer chooses the one that seems most convenient, imagining that they provide similar design solutions. Unfortunately, this is not the case. On the contrary, the two methods give very different results as was shown in this work.

The method based on nominal stiffness is excessively conservative. Moreover, this method is applicable only when the design load is small in relation to the buckling load or the column is not very slender. Very slender or heavily loaded columns should not be analysed by this method.

The method based on nominal curvature, as presented in EC2, provides good results but is unconservative for columns under sustained loads. This is due to the factor K_φ that takes into account the creep effects. This factor decreases with increasing slenderness ratio λ, indicating that creep can be neglected for very slender columns. For $\lambda \geq 52.5 + 0.75 f_{ck}$ it results $K_\varphi = 1$, indicating that creep will not be considered for these columns. The equation for this factor needs to be revised.

However, when creep effects are introduced as an additional eccentricity e_c, as adopted in CEB-FIP Model Code 1990, the method based on nominal curvature provides good results and is conservative. Therefore, this is the simplified method that should be chosen for design of reinforced concrete slender columns.

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